ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science

MATH 365
Elementary Number Theory I
FALL 2007

Final

January 18, 2008
15:00-16:50

Surname : ____________________
Name : ____________________
ID # : ____________________
Department : ____________________
Section : ____________________
Instructor : ____________________
Signature : ____________________

- The exam consists of 6 questions.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might not get full credit.
- Calculators are not allowed.

GOOD LUCK!

Please do not write below this line.

Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
20	20	20	20	20	20	120

1. Give the least complete solution to the congruence $25x \equiv 100 \pmod{35}$.

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2. Find all solutions $x, 0 < x < 500$, to

$$\begin{aligned} x &\equiv 1 \pmod{2} \\ x &\equiv 2 \pmod{3} \\ x &\equiv 3 \pmod{5} \\ x &\equiv 4 \pmod{7} \end{aligned}$$

3.
a) Give a careful statement of Euler's Theorem.
b) Is $4\,(39!) + 7!$ divisible by 41?

4.

(a) Add two negative integeres to the set $\{0, 3, 6, 9, 12, 15\}$ so that the six integers you have will form a complete residue system modulo 8. Justify your answer.

b) Does 41 divide $7 \cdot 3^{20} + 6$?

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5. Break the modulus into prime powers to find the least complete solution.

$$x^2 + x + 1 \equiv 0 \pmod{91}.$$

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6. Find all solutions to the following system of congruences.

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\begin{aligned}
x &\equiv 34 \pmod{108} \\
x &\equiv 79 \pmod{300}
\end{aligned}
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