ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science

MATH 365
Elementary Number Theory I
Second Midterm Practice Exam 2

December 17, 2007
16:40 – 18:00

In the first ten problems, compute $\sigma(n)$.

1. $n = 20$

2. $n = 30$

3. $n = 100$

4. $n = 81$

5. $n = 101$

6. $n = 40$

7. $n = 667$

8. $n = 1000$

9. $n = 272$

10. $n = 1331$

Define $s_2(n)$ to be $\sum_{d|n} d^2$. Compute $s_2(n)$ in each of the next four problems.

11. $n = 7$

12. $n = 9$

13. $n = 63$

14. $n = 1000$

Define $s_{-1}(n)$ to be $\sum_{d|n} (1/d)$. Compute $s_{-1}(n)$ in each of the next four problems.

15. Extend the table of $\sigma(n)$ to run from 31 to 50.

16. Extend the table of $\sigma(n)$ to run from 51 to 70.

17. Which of the numbers from 1 to 15 are abundant?

18. Which of the numbers from 16 to 30 are deficient?

19. Let $G(n)$ be the function defined in class. Compute $G(n)$ for n from 1 to 10.

20. Compute $G(n)$ for n from 11 to 20.

We define the function $M(n)$

$$M(n) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{if } p^2 \mid n \text{ for any prime } p \\ (-1)^k & \text{if } n = p_1 p_2 \cdots p_k \text{ where the } p\text{s are distinct primes} \end{cases}$$

21. Compute $M(n)$ for $n = 1$ to 10.
22. Compute $M(n)$ for $n = 11$ to 20.

23. Check that 1184 and 1210 are really amicable.

In problems 24 and 25, assume f is a multiplicative function with values given. Complete the table so far as you can be sure of the values.

24.

n	2	3	4	5	6	10
$f(n)$	3	4				8

25.

n	1	2	3	4	6	12
$f(n)$		4			5	

26. Prove that (defined above) is multiplicative.

27. Show that if f and g are multiplicative functions, then so is fg, where $(fg)(n) = f(n)g(n)$.

28. $F(n) = \sum_{d|n} M(d)$. Compute $F(n)$ for $n = 1, 3, 4, 6$.

29. Let F be as in the last problem. Suppose p is prime and $k > 0$. Show that $F(p^k) = 0$.

30. Let F be as in the last problem. Show that $F(n)$ is 1 and 0 otherwise.

31. Let $F(n) = \sum_{d|n} \tau(e)$. Compute $F(n)$ for $n = 1, 3, 4$, and 6.