ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science

MATH 365
Elementary Number Theory I
Second Midterm Practice Exam E

December 17, 2007
16:40 – 18:00

In the first four problems, find the order of each element in the given reduced residue system $S \pmod b$.

1. $b = 5, S = \{1, 2, 3, 4\}$

2. $b = 10, S = \{1, 3, 7, 9\}$

3. $b = 11, S = \{1, 2, \cdots, 10\}$

4. $b = 13, S = \{1, 2, \cdots, 12\}$

In the next six problems, find the order of $a \pmod b$. Compute $\phi(b)$.

5. $a = 3, b = 16$

6. $a = 5, b = 17$

7. $a = 25, b = 18$

8. $a = 18, b = 25$

9. $a = 10, b = 19$

10. $a = 4, b = 21$

In the next eight problems, compute the least residue of $a^k \pmod b$.

11. $a = 2, k = 35, b = 11$

12. $a = 3, k = 45, b = 13$

13. $a = 50, k = 40, b = 19$

14. $a = 60, k = 75, b = 19$

15. $a = 8, k = 50, b = 21$

16. $a = 9, k = 43, b = 25$

17. $a = 51, k = 100, b = 17$

18. $a = 105, k = 77, b = 53$

19. Find the last two digits of 7^{125}.

20. Find the last two digits of 9^{203}.

In the next four problems, find the least residue of $(b-1)!$

21. $b = 9$

22. $b = 10$

23. $b = 101$

24. $b = 21$

25. Prove or disprove: If $b > 1$ is composite, then $(b-1)! \equiv 0 \pmod{b}$

26. Show how the matching in the proof of Wilson's Theorem goes for $p = 13$.

27. Repeat the previous problem for $p = 17$.

28. Find the least residue of $100! + 102! \pmod{101}$

29. Find the least residue of $95! \pmod{97}$

30. Show that if $b > 1$ is not prime, then $(b-1)! \not\equiv -1 \pmod{b}$.

31. Let $a, b > 0$, and $c > 0$ be integers such that $(a, c) = 1$ and $b \mid c$. Show that the order of $a \pmod{b}$ divides the order of $a \pmod{c}$.

32. Show that no integer has order $40 \pmod{100}$.

33. Show that no integer has order $24 \pmod{72}$.