

**ÇANKAYA UNIVERSITY**  
Department of Mathematics and Computer Science

**MATH 365**  
**Elementary Number Theory I**  
First Midterm Practice Exam (B)

November 12, 2007  
16:40 – 18:00

Find the  $q$  and  $r$  guaranteed by the division algorithm for each pair  $a, b$  in problems 1 through 12.

1.  $a = 13, b = 380$

2.  $a = 15, b = 421$

3.  $a = 720, b = 155$

4.  $a = 339, b = 17$

5.  $a = 17, b = 51$

6.  $a = 21, b = 105$

7.  $a = 19, b = 0$

8.  $a = 35, b = 0$

9.  $a = 7, b = 0$

10.  $a = 9, b = -29$

11.  $a = 43, b = -500$

12.  $a = 47, b = -500$

13. What are all the common divisors of 12, and 18.

14. What are all the common divisors of 45, and 75.

15. What are all the common multiples of 4, and 6.

16. What are all the common multiples of 27, and 18.

True - False. In the next eight problems, tell which statements are true and give counterexamples for those that are false. Assume  $a, b, c$ , and  $d$  are arbitrary integers with  $a > 0$  and  $c$  and  $d$  nonzero.

17. There exist integers  $q$  and  $r$ ,  $0 \leq r < c$ , such that  $b = cq + r$ .

18. There exist integers  $q$  and  $r$ ,  $0 \leq r < |c|$ , such that  $b = cq + r$ .

19. There exist integers  $q$  and  $r$ ,  $r \leq a/2$ , such that  $b = aq + r$ .
20. There exist integers  $q$  and  $r$ ,  $r < a/2$ , such that  $b = aq + r$ .
21. The set of common divisors of  $b$  and  $c$  is the set of divisors of  $(b, c)$ .
22. The set of common multiples of  $c$  and  $b > (c, d)$ , then  $b$  is not a divisor of  $d$ .
23. If  $b$  is a multiple of  $c$ , and  $b < [c, d]$ , then  $b$  is not a multiple of  $d$ .
24. Prove that  $(a, a + 2)$  is 2 if  $a$  is even and 1 if  $a$  is odd.
25. Prove that if  $a > 0$ , then  $[a, a + 2] = a(a + 2)/2$  if  $a$  is even and  $a(a + 2)$  if  $a$  is odd.