ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science

MATH 365
Elementary Number Theory I
Final Exam
Jan 18, 2008
15:40 – 16:50

In the first 10 problems, find the least complete solution by testing.

1. $x^3 - x^2 + 3x + 1 \equiv 0 \pmod{7}$

2. $x^3 - x^2 + 3x + 1 \equiv 0 \pmod{5}$

3. $x^2 - 3x + 1001 \equiv 0 \pmod{3}$

4. $x^2 - 3x + 1001 \equiv 0 \pmod{5}$

5. $x^2 - 3x + 1001 \equiv 0 \pmod{7}$

6. $x^4 - x^3 + 5x + 1 \equiv 0 \pmod{3}$

7. $x^4 - x^3 + 5x + 1 \equiv 0 \pmod{11}$

8. $x^4 - x^3 + 5x + 1 \equiv 0 \pmod{4}$

9. $x^7 - x \equiv 0 \pmod{7}$

10. $x^7 - x \equiv 0 \pmod{5}$

In the next four problems, find all solutions x with $|x| < 10$.
11. $x^3 + x + 3 \equiv 0 \pmod{7}$

12. $x^3 + x + 3 \equiv 0 \pmod{13}$

13. $x^3 + x + 3 \equiv 0 \pmod{19}$

14. $x^3 + x + 3 \equiv 0 \pmod{3}$

In the next 10 problems, break the modulus into prime powers to find the least complete solution.
15. $x^3 - x^2 + 3x + 1 \equiv 0 \pmod{35}$

16. $x^2 - 3x + 1001 \equiv 0 \pmod{15}$

17. $x^2 - 3x + 1001 \equiv 0 \pmod{10}$

18. $x^2 - 3x + 1001 \equiv 0 \pmod{21}$

19. $x^4 - x^3 + 5x + 1 \equiv 0 \pmod{33}$

20. $x^4 - x^3 + 5x + 1 \equiv 0 \pmod{66}$

21. $x^2 + x + 1 \equiv 0 \pmod{21}$

22. $x^2 + x + 1 \equiv 0 \pmod{91}$

23. $x^2 + x + 1 \equiv 0 \pmod{273}$

24. $x^2 + x + 1 \equiv 0 \pmod{195}$

In the next four problems, a congruence is given with modulus p^2, p a prime. A solution x' to the corresponding congruence with modulus p is also given. Substitute $x = x' + py$ into the congruence and find a linear congruence with modulus p that y must satisfy as in the example we did in class. Find any corresponding solutions $x, 0 \leq x < p^2$.

25. $x^3 + 8 \equiv 0 \pmod{9}, x' = 1$

26. $x^2 + x + 1 \equiv 0 \pmod{49}, x' = 4$

27. $x^3 + 8 \equiv 0 \pmod{25}, x' = 3$

28. $x^2 + x + 1 \equiv 0 \pmod{49}, x' = 2$

In the next four problems, use the method at the end of this section to find the least complete solution.

29. $x^2 - x + 2 \equiv 0 \pmod{25}$

30. $x^2 - x + 2 \equiv 0 \pmod{49}$

31. $x^2 + x + 2 \equiv 0 \pmod{121}$

32. $x^3 + 8 \equiv 0 \pmod{49}$

In the next four problems, let k and k' be the number of elements in complete solutions to $F(x) \equiv 0 \,(\mathrm{mod}\, 3)$ and $F(x) \equiv 0 \,(\mathrm{mod}\, 9)$, respectively. Construct an integral polynomial F so that k and k' are as given.

33. $k = 3, k' = 0$
34. $k = 1, k' = 3$
35. $k = 1, k' = 1$
36. $k = 2, k' = 4$
35. Use induction on n to prove that if $m = m_1 m_2 \cdots m_n$ and $m_1, m_2, \cdots, m_n$ are positive integers, relatively prime in pairs, then $z \equiv z' \,(\mathrm{mod}\, m)$ if and only if $z \equiv z' \,(\mathrm{mod}\, m_i)$ for $i = 1, 2, \cdots, n$.