ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science

MATH 365
Elementary Number Theory I
2^{nd} Midterm
December 17, 2007
16:40-18:00

Surname : ______________________
Name : ______________________
ID # : ______________________
Department : ______________________
Section : ______________________
Instructor : ______________________
Signature : ______________________

- The exam consists of 6 questions.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might <u>not</u> get full credit.
- Calculators are <u>not</u> allowed.

GOOD LUCK!

Please do <u>not</u> write below this line.

Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
20	20	20	20	20	10	110

1.

a) Does the congruence $28x \equiv 6 \pmod{70}$ have a solution?

b) Write a complete residue system modulo 11 consisting entirely of even integers.

2. Find all solutions $z, 0 < z < 500$, to

$$\begin{aligned} z &\equiv 1 \pmod 2 \\ z &\equiv 2 \pmod 3 \\ z &\equiv 3 \pmod 5 \\ z &\equiv 4 \pmod 7 \end{aligned}$$

3.

a) Give a careful statement of Fermat's (Little) Theorem.

b) Find the least residue of

$3^{32} + 8 \pmod{227}$

4.

a) Find $1! + 2! + \cdots + 500! \pmod{189}$.

b) Give the least complete solution to the congruence $27x \equiv -18 \pmod{15}$

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5. Show that no integer has order 40 modulo 100.

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6. (Bonus) Find all solutions to the following system of congruences.

$$\begin{aligned} 5x &\equiv 2 \pmod{9} \\ 2x &\equiv 5 \pmod{13} \\ 3x &\equiv 7 \pmod{17} \end{aligned}$$