ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science

MATH 365
Elementary Number Theory I
FALL 2007

Final

January 18, 2008
15:00-16:50

Surname : ____________________
Name : ____________________
ID # : ____________________
Department : ____________________
Section : ____________________
Instructor : ____________________
Signature : ____________________

- The exam consists of 6 questions.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might <u>not</u> get full credit.
- Calculators are <u>not</u> allowed.

GOOD LUCK!

Please do <u>not</u> write below this line.

Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
20	20	20	20	20	20	120

1. Find all integer solutions to the congruence $42x \equiv 90 \pmod{156}$.

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2. Find the 2 smallest positive integers x such that

$$\begin{aligned} x &\equiv 2 \ (mod\ 7) \\ x &\equiv 3 \ (mod\ 11) \\ x &\equiv 4 \ (mod\ 13)\,. \end{aligned}$$

3.
a) Give a careful statement of Wilson's Theorem.
b) Is $4\,(29!) + 5!$ divisible by 31?

4.

(a) Add two negative integeres to the set $\{6, 11, 14, 28\}$ so that the six integers you have will form a complete residue system modulo 6. Justify your answer.

b) Does 41 divide $7 \cdot 3^{20} + 6$?

5. Break the modulus into prime powers to find the least complete solution.

$$4x^2 - 12x + 5 \equiv 0\,(mod 77)\,.$$

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6. (Bonus) Find all solutions to the following system of congruences.

$$\begin{aligned} x &\equiv 34 \ (mod\ 105) \\ x &\equiv 79 \ (mod\ 330) \end{aligned}$$