ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MATH 155 Calculus for Engineering I

Summer 2008

Problems for Recitation 1

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(p.89)

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18. $\lim_{h\to 0} \dfrac{\sqrt{5h+4}-2}{h}$

Solution:

$$\lim_{h\to 0} \frac{\sqrt{5h+4}-2}{h} = \lim_{h\to 0} \frac{\sqrt{5h+4}-2}{h} \cdot \frac{\sqrt{5h+4}+2}{\sqrt{5h+4}+2} = \lim_{h\to 0} \frac{(5h+4)-4}{h\left(\sqrt{5h+4}+2\right)}$$

$$= \lim_{h\to 0} \frac{5h}{h\left(\sqrt{5h+4}+2\right)} = \lim_{h\to 0} \frac{5}{\sqrt{5h+4}+2} = \frac{5}{\sqrt{4}+2} = \frac{5}{4}$$

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19. $\lim_{x\to 5} \dfrac{x-5}{x^2-25}$

Solution:

$$\lim_{x\to 5} \frac{x-5}{x^2-25} = \lim_{x\to 5} \frac{x-5}{(x-5)(x+5)} = \lim_{x\to 5} \frac{1}{x+5} = \frac{1}{5+5} = \frac{1}{10}$$

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20. $\lim_{x\to -3} \dfrac{x+3}{x^2+4x+3}$.

Solution:

$$\lim_{x\to -3} \frac{x+3}{x^2+4x+3} = \lim_{x\to -3} \frac{x+3}{(x+3)(x+1)} = \lim_{x\to -3} \frac{1}{x+1} = \frac{1}{-3+1} = -\frac{1}{2}.$$

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21. $\lim_{x\to -5} \dfrac{x^2+3x-10}{x+5}$.

Solution:

$$\lim_{x\to -5} \frac{x^2+3x-10}{x+5} = \lim_{x\to -5} \frac{(x+5)(x-2)}{x+5} = \lim_{x\to -5} (x-2) = -5-2 = -7$$

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22. $\lim_{x\to 2} \dfrac{x^2-7x+10}{x-2}$

Solution:

$$\lim_{x\to 2} \frac{x^2-7x+10}{x-2} = \lim_{x\to 2} \frac{(x-2)(x-5)}{x-2} = \lim_{x\to 2} (x-5) = 2-5 = -3$$

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23. $\lim_{t\to 1} \dfrac{t^2+t-2}{t^2-1}$

Solution:

$$\lim_{t\to 1}\frac{t^2+t-2}{t^2-1}=\lim_{t\to 1}\frac{(t+2)(t-1)}{(t-1)(t+1)}=\lim_{t\to 1}\frac{t+2}{t+1}=\frac{1+2}{1+1}=\frac{3}{2}.$$

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24. $\lim_{t\to -1}\frac{t^2+3t+2}{t^2-t-2}$.
Solution:
$$\lim_{t\to -1}\frac{t^2+3t+2}{t^2-t-2}=\lim_{t\to -1}\frac{(t+1)(t+2)}{(t+1)(t-2)}=\lim_{t\to -1}\frac{t+2}{t-2}=\frac{-1+2}{-1-2}=-\frac{1}{3}.$$

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25. $\lim_{x\to -2}\frac{-2x-4}{x^3+2x^2}$.
Solution:
$$\lim_{x\to -2}\frac{-2x-4}{x^3+2x^2}=\lim_{x\to -2}\frac{-2(x+2)}{x^2(x+2)}=\lim_{x\to -2}\frac{-2}{x^2}=\frac{-2}{4}=-\frac{1}{2}.$$

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26. $\lim_{y\to 0}\frac{5y^3+8y^2}{3y^4-16y^2}$.
Solution:
$$\lim_{y\to 0}\frac{5y^3+8y^2}{3y^4-16y^2}=\lim_{y\to 0}\frac{y^2(5y+8)}{y^2(3y^2-16)}=\lim_{y\to 0}\frac{5y+8}{3y^2-16}=\frac{8}{-16}=-\frac{1}{2}.$$

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27. $\lim_{u\to 1}\frac{u^4-1}{u^3-1}$.
Solution:
$$\lim_{u\to 1}\frac{u^4-1}{u^3-1}=\lim_{u\to 1}\frac{(u^2+1)(u+1)(u-1)}{(u^2+u+1)(u-1)}=\lim_{u\to 1}\frac{(u^2+1)(u+1)}{u^2+u+1}=\frac{(1+1)(1+1)}{1+1+1}=\frac{4}{3}.$$

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28. $\lim_{v\to 2}\frac{v^3-8}{v^4-16}$.
Solution:
$$\lim_{v\to 2}\frac{v^3-8}{v^4-16}=\lim_{v\to 2}\frac{(v-2)(v^2+2v+4)}{(v-2)(v+2)(v^2+4)}=\lim_{v\to 2}\frac{(v^2+2v+4)}{(v+2)(v^2+4)}=\frac{4+4+4}{(4)(8)}=\frac{12}{32}=\frac{3}{8}.$$

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29. $\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}$.
Solution:
$$\lim_{x\to 9}\frac{\sqrt{x}-3}{x-9}=\lim_{x\to 9}\frac{\sqrt{x}-3}{(\sqrt{x}-3)(\sqrt{x}+3)}=\lim_{x\to 9}\frac{1}{\sqrt{x}\sqrt{x}+3}=\frac{1}{\sqrt{9}+3}=\frac{1}{6}.$$

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30. $\lim_{x\to 4}\frac{4x-x^2}{2-\sqrt{x}}$.
Solution:
$$\lim_{x\to 4}\frac{4x-x^2}{2-\sqrt{x}}=\lim_{x\to 4}\frac{x(4-x)}{2-\sqrt{x}}=\lim_{x\to 4}\frac{x(2-\sqrt{x})(2+\sqrt{x})}{2-\sqrt{x}}=\lim_{x\to 4}x\left(2+\sqrt{x}\right)=4\left(2+\sqrt{4}\right)=16.$$

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31. $\lim_{x\to 1}\frac{x-1}{\sqrt{x+3}-2}$.
Solution:

$$\lim_{x\to 1}\frac{x-1}{\sqrt{x+3}-2}=\lim_{x\to 1}\frac{(x-1)\left(\sqrt{x+34}+2\right)}{\left(\sqrt{x+3}-2\right)\left(\sqrt{x+3}+2\right)}=\lim_{x\to 1}\frac{(x-1)\left(\sqrt{x+3}+2\right)}{x+3-4}$$
$$=\lim_{x\to 1}\frac{(x-1)\left(\sqrt{x+3}+2\right)}{x-1}=\lim_{x\to 1}\left(\sqrt{x+3}+2\right)=\sqrt{1+3}+2=4.$$

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32. $\lim_{x\to -1}\frac{\sqrt{x^2+8}-3}{x+1}$.

Solution:

$$\lim_{x\to -1}\frac{\sqrt{x^2+8}-3}{x+1}=\lim_{x\to -1}\frac{\left(\sqrt{x^2+8}-3\right)\left(\sqrt{x^2+8}+3\right)}{(x+1)\left(\sqrt{x^2+8}+3\right)}=\lim_{x\to -1}\frac{(x^2+8)-9}{(x+1)\left(\sqrt{x^2+8}+3\right)}$$
$$=\lim_{x\to -1}\frac{(x+1)(x-1)}{(x+1)\left(\sqrt{x^2+8}+3\right)}=\lim_{x\to -1}\frac{x-1}{\sqrt{x^2+8}+3}=\frac{-2}{3+3}=-\frac{1}{3}.$$

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33. $\lim_{x\to 2}\frac{\sqrt{x^2+12}-4}{x-2}$.

Solution:

$$\lim_{x\to 2}\frac{\sqrt{x^2+12}-4}{x-2}=\lim_{x\to 2}\frac{\left(\sqrt{x^2+12}-4\right)\left(\sqrt{x^2+12}+4\right)}{(x-2)\left(\sqrt{x^2+12}+4\right)}=\lim_{x\to 2}\frac{(x^2+12)-16}{(x-2)\left(\sqrt{x^2+12}+4\right)}$$
$$=\lim_{x\to 2}\frac{(x-2)(x+2)}{(x-2)\left(\sqrt{x^2+12}+4\right)}=\lim_{x\to 2}\frac{x+2}{\sqrt{x^2+12}+4}=\frac{4}{\sqrt{16}+4}=\frac{1}{2}.$$

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34. $\lim_{x\to -2}\frac{x+2}{\sqrt{x^2+5}-3}$.

Solution:

$$\lim_{x\to -2}\frac{x+2}{\sqrt{x^2+5}-3}=\lim_{x\to -2}\frac{(x+2)\left(\sqrt{x^2+5}+3\right)}{\left(\sqrt{x^2+5}-3\right)\left(\sqrt{x^2+5}+3\right)}=\lim_{x\to -2}\frac{(x+2)\left(\sqrt{x^2+5}+3\right)}{(x^2+5)-9}$$
$$=\lim_{x\to -2}\frac{(x+2)\left(\sqrt{x^2+5}+3\right)}{(x+2)(x-2)}=\lim_{x\to -2}\frac{\sqrt{x^2+5}+3}{x-2}=\frac{\sqrt{9}+3}{-4}=-\frac{3}{2}.$$

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35. $\lim_{x\to -3}\frac{2-\sqrt{x^2-5}}{x+3}$

Solution:

$$\lim_{x\to -3}\frac{2-\sqrt{x^2-5}}{x+3}=\lim_{x\to -3}\frac{\left(2-\sqrt{x^2-5}\right)\left(2+\sqrt{x^2-5}\right)}{(x+3)\left(2+\sqrt{x^2-5}\right)}=\lim_{x\to -3}\frac{4-(x^2-5)}{(x+3)\left(2+\sqrt{x^2-5}\right)}$$
$$=\lim_{x\to -3}\frac{9-x^2}{(x+3)\left(2+\sqrt{x^2-5}\right)}=\lim_{x\to -3}\frac{(3-x)(3+x)}{(x+3)\left(2+\sqrt{x^2-5}\right)}=\lim_{x\to -3}\frac{3-x}{2+\sqrt{x^2-5}}$$
$$=\frac{6}{2+\sqrt{4}}=\frac{3}{2}.$$

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36. $\lim_{x\to 4}\frac{4-x}{5-\sqrt{x^2+9}}$

Solution:

$$\lim_{x\to 4}\frac{4-x}{5-\sqrt{x^2+9}}=\lim_{x\to 4}\frac{(4-x)\left(5+\sqrt{x^2+9}\right)}{\left(5-\sqrt{x^2+9}\right)\left(5+\sqrt{x^2+9}\right)}=\lim_{x\to 4}\frac{(4-x)\left(5+\sqrt{x^2+9}\right)}{\left(5-\sqrt{x^2+9}\right)\left(5+\sqrt{x^2+9}\right)}$$
$$=\lim_{x\to 4}\frac{(4-x)\left(5+\sqrt{x^2+9}\right)}{25-(x^2+9)}=\lim_{x\to 4}\frac{(4-x)\left(5+\sqrt{x^2+9}\right)}{16-x^2}=\lim_{x\to 4}\frac{(4-x)\left(5+\sqrt{x^2+9}\right)}{(4-x)(4+x)}$$

$$= \lim_{x\to 4} \frac{\left(5+\sqrt{x^2+9}\right)}{4+x} = \frac{5+\sqrt{25}}{8} = \frac{5}{4}$$

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1. Finding One-Sided Limits Algebraically

(p.113)

11. $\displaystyle\lim_{x\to -0.5^-} \sqrt{\frac{x+2}{x+1}}$

Solution:

$$\lim_{x\to -0.5^-} \sqrt{\frac{x+2}{x+1}} = \sqrt{\frac{-0.5+2}{-0.5+1}} = \sqrt{\frac{3/2}{1/2}} = \sqrt{3}$$

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15. $\displaystyle\lim_{h\to 0^+} \frac{\sqrt{h^2+4h+5}-\sqrt{5}}{h}$

Solution:

$$\lim_{h\to 0^+} \frac{\sqrt{h^2+4h+5}-\sqrt{5}}{h} = \lim_{h\to 0^+} \left(\frac{\sqrt{h^2+4h+5}-\sqrt{5}}{h}\right)\left(\frac{\sqrt{h^2+4h+5}+\sqrt{5}}{\sqrt{h^2+4h+5}+\sqrt{5}}\right)$$

$$= \lim_{h\to 0^+} \frac{(h^2+4h+5)-5}{h\left(\sqrt{h^2+4h+5}+\sqrt{5}\right)} = \lim_{h\to 0^+} \frac{h\,(h+4)}{h\left(\sqrt{h^2+4h+5}+\sqrt{5}\right)} = \frac{0+4}{\sqrt{5}+\sqrt{5}} = \frac{2}{\sqrt{5}}$$

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16. $\displaystyle\lim_{h\to 0^-} \frac{\sqrt{6}-\sqrt{5h^2+11h+6}}{h}$

Answer: $-\dfrac{11}{2\sqrt{2}}$

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30. $\displaystyle\lim_{x\to 0} \frac{x^2-x+\sin x}{2x}$

Solution:

$$\lim_{x\to 0} \frac{x^2-x+\sin x}{2x} = \lim_{x\to 0} \left(\frac{x}{2}-\frac{1}{2}+\frac{1}{2}\left(\frac{\sin x}{x}\right)\right) = 0-\frac{1}{2}+\frac{1}{2}(1) = 0.$$

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31. $\displaystyle\lim_{t\to 0} \frac{\sin(1-\cos t)}{1-\cos t}$

Solution:

$$\lim_{t\to 0} \frac{\sin(1-\cos t)}{1-\cos t} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1 \text{ since } \theta = 1-\cos t \to 0 \text{ as } t\to 0.$$

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32. $\displaystyle\lim_{h\to 0} \frac{\sin(\sinh)}{\sinh}$

Solution:

$$\lim_{h\to 0} \frac{\sin(\sinh)}{\sinh} = \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1 \text{ since } \theta = \sinh \to 0 \text{ as } h\to 0.$$

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33. $\displaystyle\lim_{\theta\to 0} \frac{\sin\theta}{\sin 2\theta}$

Solution:

$$\lim_{\theta\to 0}\frac{\sin\theta}{\sin 2\theta} = \lim_{\theta\to 0}\left(\frac{\sin\theta}{\sin 2\theta}\cdot\frac{2\theta}{2\theta}\right) = \frac{1}{2}\lim_{\theta\to 0}\left(\frac{\sin\theta}{\theta}\cdot\frac{2\theta}{\sin 2\theta}\right) = \frac{1}{2}\cdot 1\cdot 1 = \frac{1}{2}$$

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34. $\lim_{x\to 0}\frac{\sin 5x}{\sin 4x}$

Solution:

$$\lim_{x\to 0}\frac{\sin 5x}{\sin 4x} = \lim_{x\to 0}\left(\frac{\sin 5x}{\sin 4x}\cdot\frac{4x}{5x}\cdot\frac{5}{4}\right)$$

$$= \lim_{x\to 0}\left(\frac{\sin 5x}{5x}\cdot\frac{4x}{\sin 4x}\right) = \frac{5}{4}\cdot 1\cdot 1 = \frac{5}{4}$$

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35. $\lim_{x\to 0}\frac{\tan 3x}{\sin 8x}$

Solution:

$$\lim_{x\to 0}\frac{\tan 3x}{\sin 8x} = \lim_{x\to 0}\left(\frac{\sin 3x}{\cos 3x}\cdot\frac{1}{\sin 8x}\right) = \lim_{x\to 0}\left(\frac{\sin 3x}{\cos 3x}\cdot\frac{1}{\sin 8x}\cdot\frac{8x}{3x}\cdot\frac{3}{8}\right)$$

$$= \frac{3}{8}\lim_{x\to 0}\left(\frac{1}{\cos 3x}\right)\left(\frac{\sin 3x}{3x}\right)\left(\frac{8x}{\sin 8x}\right) = \frac{3}{8}\cdot 1\cdot 1\cdot 1 = \frac{3}{8}$$

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36. $\lim_{y\to 0}\frac{\sin 3y\cot 5y}{y\cot 4y}$

Solution:

$$\lim_{y\to 0}\frac{\sin 3y\cot 5y}{y\cot 4y} = \lim_{y\to 0}\frac{\sin 3y\sin 4y\cos 5y}{y\cos 4y\sin 5y} = \lim_{y\to 0}\left(\frac{\sin 3y}{y}\right)\left(\frac{\sin 4y}{\cos 4y}\right)\left(\frac{\cos 5y}{\sin 5y}\right)\left(\frac{3\cdot 4\cdot 5y}{3\cdot 4\cdot 5y}\right)$$

$$= \lim_{y\to 0}\left(\frac{\sin 3y}{3y}\right)\left(\frac{\sin 4y}{4y}\right)\left(\frac{5y}{\sin 5y}\right)\left(\frac{\cos 5y}{\cos 4y}\right)\left(\frac{3\cdot 4}{5}\right) = 1\cdot 1\cdot 1\cdot 1\cdot\frac{12}{5} = \frac{12}{5}$$

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2. Calculating Limits as $x \to \pm\infty$

43. $\lim_{x\to\infty}\frac{\sin 2x}{x}$

Solution:

$$-\frac{1}{x} \le \frac{\sin 2x}{x} \le \frac{1}{x} \implies \lim_{x\to 0}\frac{\sin 2x}{x} = 0 \text{ by the Sandwich Theorem.}$$

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44. $\lim_{\theta\to -\infty}\frac{\cos\theta}{3\theta}$

Solution:

$$-\frac{1}{3\theta} \le \frac{\cos\theta}{3\theta} \le \frac{1}{3\theta} \implies \lim_{\theta\to -\infty}\frac{\cos\theta}{3\theta} = 0 \text{ by the Sandwich Theorem.}$$

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45. $\lim_{t\to\infty}\frac{2-t+\sin t}{t+\cos t}$

Solution:

$$\lim_{t\to\infty}\frac{2-t+\sin t}{t+\cos t} = \lim_{t\to\infty}\frac{\frac{2}{t}-1+\left(\frac{\sin t}{t}\right)}{1+\frac{\cos t}{t}} = \frac{0-1+0}{1+0} = -1.$$

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46. $\lim_{r\to\infty}\frac{r+\sin r}{2r+7-5\sin r}$

Solution:
$$\lim_{r\to\infty} \frac{r+\sin r}{2r+7-5\sin r} = \lim_{r\to\infty} \frac{1+\left(\frac{\sin r}{r}\right)}{2+\frac{7}{r}-5\left(\frac{\sin r}{r}\right)} = \frac{1+0}{2+0-0} = \frac{1}{2}$$