ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MATH 155 Calculus for Engineering I

Summer 2008

Problems and Solutions for Recitation 2

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Find the first and second derivatives of the functions in Exercises 31-38.

31. $y = \dfrac{x^3+7}{x}$

Solution:

$y = \dfrac{x^3+7}{x} = x^2 + \dfrac{7}{x} \Longrightarrow y' = 2x - \dfrac{7}{x^2}, y'' = 2 + \dfrac{14}{x^3}.$

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32. $s = \dfrac{t^2+5t-1}{t^2}$

Solution:

$s = \dfrac{t^2+5t-1}{t^2} = 1 + \dfrac{5}{t} - \dfrac{1}{t^2} \Longrightarrow \dfrac{ds}{dt} = -\dfrac{5}{t^2} + \dfrac{2}{t^3}, \dfrac{d^2s}{dt^2} = \dfrac{10}{t^3} - \dfrac{6}{t^4}.$

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33. $r = \dfrac{(\theta-1)\left(\theta^2+\theta+1\right)\sin\theta}{\theta^3}$

Solution:

$r = \dfrac{\left(\theta^3-1\right)\sin\theta}{\theta^3} = \left(1 - \dfrac{1}{\theta^3}\right)\sin\theta = \sin\theta - \theta^{-3}\sin\theta \Longrightarrow \dfrac{dr}{d\theta} = \cos\theta + 3\theta^{-4}\sin\theta - \theta^{-3}\cos\theta$

$\dfrac{d^2r}{d\theta^2} = -\sin\theta - 12\theta^{-5}\sin\theta + 3\theta^{-4}\cos\theta + 3\theta^{-4}\cos\theta + \theta^{-3}\sin\theta$

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34. $u = \dfrac{(x^2+x)(x^2-x+1)}{x^4}$

Solution:

$u = \dfrac{(x^2+x)(x^2-x+1)}{x^4} = \dfrac{(x+1)(x^2-x+1)}{x^3} = \dfrac{x^3+1}{x^3} = 1 + \dfrac{1}{x^3} \Longrightarrow \dfrac{du}{dx} = -\dfrac{3}{x^4}$

$\Longrightarrow \dfrac{d^2u}{dx^2} = \dfrac{12}{x^5}$

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35. $w = \left(\dfrac{1+3z}{3z}\right)(3-z)$

Solution:

$w = \left(\dfrac{1+3z}{3z}\right)(3-z) = \left(\dfrac{1}{3z} + 1\right)(3-z) = \dfrac{1}{z} - \dfrac{1}{3} + 3 - z \Longrightarrow \dfrac{dw}{dz} = -\dfrac{1}{z^2} - 1$

$\Longrightarrow \dfrac{d^2w}{dz^2} = \dfrac{2}{z^3}$

36. $w=(z+1)(z-1)\left(z^2+1\right)$

Solution:

$w=(z+1)(z-1)\left(z^2+1\right)=z^4-1\Longrightarrow\dfrac{dw}{dz}=4z^3\Longrightarrow\dfrac{d^2w}{dz^2}=12z^2$

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37. $p=\left(\dfrac{q^3+3}{12q}\right)\left(\dfrac{q^4-1}{q^3}\right)$

Solution:

$p=\left(\dfrac{q^3+3}{12q}\right)\left(\dfrac{q^4-1}{q^3}\right)=\left(\dfrac{q^2}{12}+\dfrac{1}{4q}\right)\left(q-\dfrac{1}{q^3}\right)=\dfrac{q^3}{12}-\dfrac{1}{12q}+\dfrac{1}{4}-\dfrac{1}{4q^4}$

$\dfrac{dp}{dq}=\dfrac{q^2}{4}+\dfrac{1}{12q^2}+\dfrac{1}{q^5}\Longrightarrow\dfrac{d^2p}{dq^2}=\dfrac{q}{2}-\dfrac{1}{6q^3}-\dfrac{5}{q^6}$

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38. $p=\dfrac{q^2+3}{(q-1)^3+(q+1)^3}$

Solution:

$p=\dfrac{q^2+3}{(q-1)^3+(q+1)^3}=p=\dfrac{q^2+3}{(q^3-3q^2+3q-1)+(q^3+3q^2+3q+1)}=\dfrac{q^2+3}{2q^3+6q}=\dfrac{q^2+3}{2q\left(q^2+3\right)}=$
$\dfrac{1}{2q}=\dfrac{1}{2}q^{-1}$

$\Longrightarrow\dfrac{dp}{dq}=-\dfrac{1}{2}q^{-2}=-\dfrac{1}{2q^2}\Longrightarrow\dfrac{d^2p}{dq^2}=q^{-3}=\dfrac{1}{q^3}$

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39. Suppose u and v are functions of x that are differentiable at $x=0$.

$$u(0)=5,u'(0)=-3,v(0)=-1,v'(0)=2.$$

Find the values of the following derivatives at $x=0$.

a. $\dfrac{d}{dx}(uv)$ **b.** $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)$, **c.** $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)$, **d.** $\dfrac{d}{dx}(7v-2u)$

Solution:

(a) $\dfrac{d}{dx}(uv)=uv'+vu'\Longrightarrow\dfrac{d}{dx}(uv)\mid_{x=0}=u(0)v'(0)+v(0)u'(1)=5\cdot2+(-1)(-3)=13$

(b) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{vu'-uv'}{v^2}\Longrightarrow\dfrac{d}{dx}\left(\dfrac{u}{v}\right)\mid_{x=0}=\dfrac{v(0)u'(0)-u(0)v'(0)}{(v(0))^2}=\dfrac{(-1)(-3)-(5)(2)}{(-1)^2}=$
-7

(c) $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)=\dfrac{uv-vu'}{u^2}\Longrightarrow\dfrac{d}{dx}\left(\dfrac{v}{u}\right)\mid_{x=0}=\dfrac{u(0)v'(0)-v(0)u'(0)}{(u(0))^2}=\dfrac{(5)(2)-(-1)(-3)-5\cdot0}{(5)^2}=$
$\dfrac{7}{25}$

(d) $\dfrac{d}{dx}(7v-2u)=7v'-2u'\Longrightarrow\dfrac{d}{dx}(7v-2u)\mid_{x=0}=7v'(0)-2u'(0)=7\cdot2-2(-3)=20.$

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40. Suppose u and v are differentiable functions of x and that

$$u(1)=2,u'(1)=0,v(1)=5,v'(1)=-1.$$

Find the values of the following derivatives at $x=1$.

a. $\dfrac{d}{dx}(uv)$ **b.** $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)$, **c.** $\dfrac{d}{dx}\left(\dfrac{v}{u}\right)$, **d.** $\dfrac{d}{dx}(7v-2u)$

Solution:

(a) $\frac{d}{dx}(uv)\,|_{x=1}= u(1)\,v'(1) + v(1)\,u'(1) = 2\cdot(-1) + 5\cdot 0 = -2$

(b) $\frac{d}{dx}\left(\frac{u}{v}\right)|_{x=1}= \frac{v(1)\,u'(1) - u(1)\,v'(1)}{(v(1))^2} = \frac{5\cdot 0 - 2\cdot(-1)}{(5)^2} = \frac{2}{25}$

(c) $\frac{d}{dx}\left(\frac{v}{u}\right)|_{x=1}= \frac{u(1)\,v'(1) - v(1)\,u'(1)}{(u(1))^2} = \frac{2\cdot(-1) - 5\cdot 0}{(2)^2} = -\frac{1}{2}$

(d) $\frac{d}{dx}(7v - 2u)\,|_{x=1}= 7v'(1) - 2u'(1) = 7(-1) - 2\cdot 0 = -7.$

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1. The Chain Rule

(p.201)
Find the derivatives.

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37. $q = \sin\left(\frac{t}{\sqrt{t+1}}\right)$

Solution:

$$q = \sin\left(\frac{t}{\sqrt{t+1}}\right) \Longrightarrow \frac{dq}{dt} = \cos\left(\frac{t}{\sqrt{t+1}}\right)\cdot\frac{d}{dt}\left(\frac{t}{\sqrt{t+1}}\right) = \cos\left(\frac{t}{\sqrt{t+1}}\right)\cdot\frac{\sqrt{t+1}\,(1) - t\frac{d}{dt}\left(\sqrt{t+1}\right)}{\left(\sqrt{t+1}\right)^2}$$

$$= \cos\left(\frac{t}{\sqrt{t+1}}\right)\frac{\sqrt{t+1} - \frac{1}{2\sqrt{t+1}}}{t+1} = \left(\frac{t+2}{2\,(t+1)^{3/2}}\right)\cos\left(\frac{t}{\sqrt{t+1}}\right)$$

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38. $q = \cot\left(\frac{\sin t}{t}\right)$

Solution:

$$q = \cot\left(\frac{\sin t}{t}\right) \Longrightarrow \frac{dq}{dt} = -\csc^2\left(\frac{\sin t}{t}\right)\cdot\frac{d}{dt}\left(\frac{\sin t}{t}\right) = \left(-\csc^2\left(\frac{\sin t}{t}\right)\right)\left(\frac{t\cos t - \sin t}{t^2}\right)$$

Find $\frac{dy}{dt}$

41. $y = (1+\cos 2t)^{-4}$

Solution:

$$y = (1+\cos 2t)^{-4} \Longrightarrow \frac{dy}{dt} = -4\,(1-\cos 2t)^{-5}\cdot\frac{d}{dt}(1+\cos 2t) = \frac{8\sin 2t}{(1+\cos 2t)^5}$$

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47. $y = \sqrt{1+\cos\left(t^2\right)}$

Solution:

$$y = \left(1+\cos\left(t^2\right)\right)^{1/2} \Longrightarrow \frac{dy}{dt} = -\frac{t\sin t^2}{\sqrt{1+\cos\left(t^2\right)}}$$

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48. $y = 4\sin\left(\sqrt{1+\sqrt{t}}\right)$

Solution:

$$y = 4\sin\left(\sqrt{1+\sqrt{t}}\right) \Longrightarrow \frac{dy}{dt} = \frac{\cos\left(\sqrt{1+\sqrt{t}}\right)}{\sqrt{t}\sqrt{1+\sqrt{t}}}$$

2. Implicit Differentiation

(p.211)
Find dy/dx
19. $x^2y + xy^2 = 6$
Solution:
Step 1: $\left(x^2\dfrac{dy}{dx} + y \cdot 2x\right) + \left(x \cdot 2y\dfrac{dy}{dx} + y^2 \cdot 1\right) = 0$
Step 2: $x^2\dfrac{dy}{dx} + 2xy\dfrac{dy}{dx} = -2xy - y^2$
Step 3: $\dfrac{dy}{dx}\left(x^2 + 2xy\right) = -2xy - y^2$
Step 4: $\dfrac{dy}{dx} = \dfrac{-2xy - y^2}{x^2 + 2xy}$

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20. $x^3 + y^3 = 18xy$
Solution:
$x^3 + y^3 = 18xy \Longrightarrow 3x^2 + 3y^2\dfrac{dy}{dx} = 18y + 18x\dfrac{dy}{dx} \Longrightarrow \left(3y^2 - 18x\right)\dfrac{dy}{dx} = 18y - 3x^2$
$\Longrightarrow \dfrac{dy}{dx} = \dfrac{6y - x^2}{y^2 - 6x}$

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21. $2xy + y^2 = x + y$
Solution:
Step 1: $\left(2x\dfrac{dy}{dx} + 2y\right) + 2y\dfrac{dy}{dx} = 1 + \dfrac{dy}{dx}$
Step 2: $2x\dfrac{dy}{dx} + 2y\dfrac{dy}{dx} - \dfrac{dy}{dx} = 1 - 2y$
Step 3: $\dfrac{dy}{dx}(2x + 2y - 1) = 1 - 2y$
Step 4: $\dfrac{dy}{dx} = \dfrac{1 - 2y}{2x + 2y - 1}$.

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22. $x^3 - xy + y^3 = 1$
Solution:
$x^3 - xy + y^3 = 1 \Longrightarrow 3x^2 - y - x\dfrac{dy}{dx} + 3y^2\dfrac{dy}{dx} = 0 \Longrightarrow \left(3y^2 - x\right)\dfrac{dy}{dx} = y - 3x^2$
$\Longrightarrow \dfrac{dy}{dx} = \dfrac{y - 3x^2}{3y^2 - x}$

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23. $x^2(x - y)^2 = x^2 - y^2$
Solution:
Step 1: $x^2\left[2(x - y)\left(1 - \dfrac{dy}{dx}\right)\right] + (x - y)^2 2x = 2x - 2y\dfrac{dy}{dx}$
Step 2: $2x\dfrac{dy}{dx} + 2y\dfrac{dy}{dx} - \dfrac{dy}{dx} = 1 - 2y$
Step 3: $\dfrac{dy}{dx}\left[-2x^2(x - y) + 2y\right] = 2x\left[1 - x(x - y) - \left(x - y^2\right)\right]$

Step 4: $\dfrac{dy}{dx} = \dfrac{2x\left[1 - x(x-y) - (x-y^2)\right]}{-2x^2(x-y) + 2y} = \dfrac{x - 2x^3 + 3x^2y - xy^2}{x^2y - x^3 + y}$.

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31. $y \sin\left(\dfrac{1}{y}\right) = 1 - xy$

Solution:

$y\left[\cos\left(\dfrac{1}{y}\right) \cdot (-1)\dfrac{1}{y^2} \cdot \dfrac{dy}{dx}\right] + \sin\left(\dfrac{1}{y}\right) \cdot \dfrac{dy}{dx} = -x\dfrac{dy}{dx} - y$

$\Longrightarrow \dfrac{dy}{dx}\left[-\dfrac{1}{y}\cos\left(\dfrac{1}{y}\right) + \sin\left(\dfrac{1}{y}\right) + x\right] = -y$

$\Longrightarrow \dfrac{dy}{dx} = \dfrac{-y}{-\frac{1}{y}\cos\left(\frac{1}{y}\right) + \sin\left(\frac{1}{y}\right) + x}$

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32. $y^2 \cos\left(\dfrac{1}{y}\right) = 2x + 2y$

Solution:

$y^2\left[-\sin\left(\dfrac{1}{y}\right) \cdot (-1)\dfrac{1}{y^2} \cdot \dfrac{dy}{dx}\right] + \cos\left(\dfrac{1}{y}\right) \cdot 2y\dfrac{dy}{dx} = 2 + 2\dfrac{dy}{dx}$

$\Longrightarrow \dfrac{dy}{dx}\left[\sin\left(\dfrac{1}{y}\right) + 2y\cos\left(\dfrac{1}{y}\right) - 2\right] = 2$

$\Longrightarrow \dfrac{dy}{dx} = \dfrac{2}{\sin\left(\frac{1}{y}\right) + 2y\cos\left(\frac{1}{y}\right) - 2}$

Find $dr/d\theta$.

33. $\theta^{1/2} + r^{1/2} = 1$

Solution:

$\theta^{1/2} + r^{1/2} = 1 \Longrightarrow \dfrac{1}{2}\theta^{-1/2} + \dfrac{1}{2}r^{-1/2} \cdot \dfrac{dr}{d\theta} = 0$

$\Longrightarrow \dfrac{dr}{d\theta}\left[\dfrac{1}{r\sqrt{r}}\right] = \dfrac{-1}{2\sqrt{\theta}}$

$\Longrightarrow \dfrac{dr}{d\theta} = -\dfrac{2\sqrt{r}}{2\sqrt{\theta}} = -\dfrac{\sqrt{r}}{\sqrt{\theta}}$

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36. $\cos r + \cot\theta = r\theta$

Solution:

$\cos r + \cot\theta = r\theta \Longrightarrow (-\sin r)\dfrac{dr}{d\theta} - \csc^2\theta = r + \theta\dfrac{dr}{d\theta}$

$\Longrightarrow \dfrac{dr}{d\theta}[-\sin r - \theta] = r + \csc^2\theta$

$\Longrightarrow \dfrac{dr}{d\theta} = \dfrac{r + \csc^2\theta}{-\sin r - \theta}$

Find d^2y/dx^2.

37. $x^2 + y^2 = 1$

Solution:

$x^2 + y^2 = 1 \Longrightarrow 2x + 2yy' = 0$

$\Longrightarrow 2yy' = -2x \Longrightarrow \dfrac{dy}{dx} = y' = -\dfrac{x}{y};$

now to find d^2y/dx^2, $\dfrac{d}{dx}(y') = \dfrac{d}{dx}\left(-\dfrac{x}{y}\right) \Longrightarrow y'' = \dfrac{y(-1) + xy'}{y^2} = \dfrac{-y + x\left(-\frac{x}{y}\right)}{y^2}$

since $y' = -\dfrac{x}{y} \Longrightarrow \dfrac{d^2y}{dx^2} = y'' = \dfrac{-y^2 - x^2}{y^3} = \dfrac{-y^2 - (1 - y^2)}{y^3} = \dfrac{-1}{y^3}.$

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38. $x^{2/3} + y^{2/3} = 1$

Answer:

$\dfrac{d^2y}{dx^2} = y'' = \dfrac{y^{1/3}}{4x^{4/3}} + \dfrac{1}{3y^{1/3}x^{2/3}}$

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39. $y^2 = x^2 + 2x$

Answer:

$\dfrac{dy}{dx} = y' = \dfrac{x+1}{y}$

$\dfrac{d^2y}{dx^2} = y'' = \dfrac{y^2 - (x+1)^2}{y^3}$

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43. If $x^3 + y^3 = 16$, find the value of $\dfrac{d^2y}{dx^2}$ at the point $(2, 2)$

Answer:

$y' = -\dfrac{x^2}{y^2}$

$y'' = \dfrac{-2xy^3 - 2x^4}{y^5}$

$y''\,|_{(2,2)} = -2$

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44. If $xy + y^2 = 1$, find the value of $\dfrac{d^2y}{dx^2}$ at the point $(0, -1)$

Solution:

$xy + y^2 = 1 \Longrightarrow xy' + y + 2yy' = 0 \Longrightarrow y'(x + 2y) = -y$

$\Longrightarrow y' = \dfrac{-y}{x + 2y} \Longrightarrow y'' = \dfrac{(x + 2y)(-y') - (-y)(1 + 2y')}{(x + 2y)^2};$

since $y'\,|_{(0,-1)} = -\dfrac{1}{2}$ we obtain $y''\,|_{(0,-1)} = \dfrac{(-2)\left(\frac{1}{2}\right) - (1)(0)}{4} = -\dfrac{1}{4}$

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45. Find the slope of the curve $y^2 + x^2 = y^4 - 2x$ at $(-2, 1)$ and $(-2, -1)$.

Solution:

$y^2 + x^2 = y^4 - 2x$ at $(-2, 1)$ and $(-2, -1) \Longrightarrow 2yy' + 2x = 4y^3y' - 2$

$\Longrightarrow 2yy' - 4y^3y' = -2 - 2x$

$\Longrightarrow y'\left(2y - 4y^3\right) = -2 - 2x$

$$y' = \frac{-2-2x}{2y-4y^3} = \frac{x+1}{2y^3-y}$$

$y' \mid_{(-2,1)} = -1$ and $y' \mid_{(-2,-1)} = 1$.

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69. The line that is normal to the curve $x^2+2xy-3y^2=0$ at $(1,1)$ intersects the curve at what other point?

Solution:

$x^2+2xy-3y^2=0 \Longrightarrow 2x+2xy'+2y-6yy'=0 \Longrightarrow y'(2x-6y) = -2x-2y \Longrightarrow y' = \dfrac{x+y}{3y-x} \Longrightarrow$ the slope of the tangent line

$m = y' \mid_{(1,1)} = \dfrac{x+y}{3y-x} \mid_{(1,1)} = 1 \Longrightarrow$ the equation of the normal line at $(1,1)$ is

$y-1 = -1(x-1) \Longrightarrow y = -x+2$.

To find where the normal line intersects the curve we substitute into its equation:

$x^2+2x(2-x)-3(2-x)^2 = 0 \Longrightarrow x^2+4x-2x^2-3\left(4-4x+x^2\right) = 0$

$\Longrightarrow -4x^2+16x-12 = 0 \Longrightarrow x^2-4x+3 = 0 \Longrightarrow (x-3)(x-1) = 0 \Longrightarrow x = 3$ and $y = -x+2 = -1$.

Therefore, the normal to the curve at $(1,1)$ intersects the curve at the point $(3,-1)$.

Note that it also intersects the curve at $(1,1)$.

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70. Find the normals to the curve $xy+2x-y=0$ that are parallel to the line $2x+y=0$.

Solution:

$xy+2x-y=0 \Longrightarrow x\dfrac{dy}{dx}+y+2-\dfrac{dy}{dx} = 0 \Longrightarrow \dfrac{dy}{dx} = \dfrac{y+2}{1-x}$; the slope of the line $2x+y=0$ is -2.

In order to be parallel, the normal lines also have slope of -2.

Since a normal is perpendicular to a tangent, the slope of the tangent is $\dfrac{1}{2}$.

Therefore, $\dfrac{y+2}{1-x} = \dfrac{1}{2} \Longrightarrow 2y+4 = 1-x \Longrightarrow x = -3-2y$.

Substituting in the original equation, $y(-3-2y)+2(-3-2y)-y = 0 \Longrightarrow y^2+4y+3 = 0 \Longrightarrow y = -3$ or $y = -1$.

If $y = -3$, then $x = 3$ and $y+3 = -2(x-3) \Longrightarrow y = -2x+3$.

If $y = -1$, then $x = -1$ and $y+1 = -2(x+1) \Longrightarrow y = -2x-3$.

(p.237)

68. For what values of the constant m, if any, is

$$f(x) = \begin{cases} \sin 2x & if \quad x \le 0 \\ mx & if \quad x > 0 \end{cases}$$

a. continuous at $x=0$?

b. differentiable at $x=0$?

Give reasons to your answers.

Solution:

(a) $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} \sin 2x = 0$ and $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} mx = 0 \Longrightarrow \lim\limits_{x\to 0} f(x)$ is independent of m; since $f(0) = 0 = \lim\limits_{x\to 0} f(x)$ it follows that f is continuous at $x=0$ for all values of m.

(b) $\lim_{x\to 0^-} f'(x) = \lim_{x\to 0^-} (\sin 2x)' = \lim_{x\to 0^-} 2\cos x$ and $\lim_{x\to 0^+} f'(x) = \lim_{x\to 0^+} (mx)' = \lim_{x\to 0^+} m = m \Longrightarrow$ f is differentiable at $x = 0$ provided that $\lim_{x\to 0^-} f'(x) = \lim_{x\to 0^+} f'(x) \Longrightarrow m = 2.$

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71. Find the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ where the tangent is parallel to the x-axis.
Solution:
$y = 2x^3 - 3x^2 - 12x + 20 \Longrightarrow \dfrac{dy}{dx} = 6x^2 - 6x - 12$; the tangent is parallel to the x-axis when $\dfrac{dy}{dx} = 0 \Longrightarrow 6x^2 - 6x - 12 = 0 \Longrightarrow x^2 - x - 2 = 0 \Longrightarrow (x-2)(x+1) = 0 \Longrightarrow x = 2$ or $x = -1$ $\Longrightarrow (2, 0)$ and $(-1, 27)$ are points on the curve where the tangent is parallel to the x-axis.

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3. Extreme Values of Functions

(p.253)
15. Find the absolute maximum and minimum values of $f(x) = \dfrac{2}{3}x - 5$ on the interval $-2 \le x \le 3$.
Solution:
$f(x) = \dfrac{2}{3}x - 5 \Longrightarrow f'(x) = \dfrac{2}{3} \Longrightarrow$ no critical points; $f(-2) = -\dfrac{19}{3}$, $f(3) = -3 \Longrightarrow$ the absolute maximum is -3 at $x = 3$ and the absolute minimum is $-\dfrac{19}{3}$ at $x = -2$.

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16. Find the absolute maximum and minimum values of $f(x) = -x - 4$ on the interval $-4 \le x \le 1$.
Answer:
the absolute maximum is 0 at $x = -4$ and the absolute minimum is -5 at $x = 1$.

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19. Find the absolute maximum and minimum values of $F(x) = -\dfrac{1}{x^2}$ on the interval $0.5 \le x \le 2$.
Solution:
$F(x) = -\dfrac{1}{x^2} = -x^{-2} \Longrightarrow F'(x) = 2x^{-3} = \dfrac{2}{x^3}$, however $x = 0$ is not a critical point since 0 is not in the domain; $F(0.5) = -4, F(2) = -0.25 \Longrightarrow$ the absolute maximum is -0.25 at $x = 2$ and the absolute minimum is -4 at 0.5.

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20. Find the absolute maximum and minimum values of $F(x) = -\dfrac{1}{x}$ on the interval $-2 \le x \le -1$.
Answer:
the absolute maximum is 1 at $x = -1$ and the absolute minimum is $\dfrac{1}{2}$ at $x = -2$.

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21. Find the absolute maximum and minimum values of $h(x) = \sqrt[3]{x}$ on the interval $-1 \le x \le 8$.
Answer:
the absolute maximum is 2 at $x = 8$ and the absolute minimum is -1 at $x = -1$.

22. Find the absolute maximum and minimum values of $h(x) = -3x^{2/3}$ on the interval $-1 \le x \le 1$.
Answer:
the absolute maximum is 0 at $x = 0$ and the absolute minimum is -3 at $x = 1, x = -1$.

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23. Find the absolute maximum and minimum values of $g(x) = \sqrt{4 - x^2}$ on the interval $-2 \le x \le 1$.
Solution:
$g(x) = \sqrt{4 - x^2} = \left(4 - x^2\right)^{1/2} \Longrightarrow g'(x) = \frac{1}{2}\left(4 - x^2\right)^{-1/2}(-2x) = \frac{-x}{\sqrt{4 - x^2}} \Longrightarrow$ critical points at $x = -2$ and $x = 0$, but not at $x = 2$ because 2 is not in the domain; $g(-2) = 0, g(0) = 2$, $g(1) = \sqrt{3} \Longrightarrow$ the absolute maximum is 2 at $x = 0$ and the absolute minimum is 0 at $x = -2$.

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45. Determine the local extreme values of $y = x^{2/3}(x + 2)$.
Solution:
$y' = \frac{5}{3}x^{\frac{2}{3}} + \frac{4}{3}x^{-\frac{1}{3}}$. The critical points are $x = -\frac{4}{5}$ and $x = 0$; $y(0) = 0, y\left(-\frac{4}{5}\right) = 1.034$. So the local maximum is $1,304$ at $-\frac{4}{5}$ and the local minimum is 0 at $x = 0$.