ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MATH 155 Calculus for Engineering I

Summer 2008

Problems and Solutions for Recitation 3

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9:40

Use the steps of graphing procedure to graph the equations in Exercises 9-40. Include the coordinates of any local extreme points and inflection points.

9. $y = x^2 - 4x + 3$

Solution:

When $y = x^2 - 4x + 3$, then $y' = 2x - 4 = 2(x-2)$ and $y'' = 2$. The curve rises on $(2, \infty)$ and falls on $(-\infty, 2)$. At $x = 2$ there is a minimum. Since $y'' > 0$, the curve is concave up for all x.

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10. $y = 6 - 2x - x^2$

Solution:

When $y = 6 - 2x - x^2$, then $y' = -2 - 2x = -2(1+x)$ and $y'' = -2$. The curve rises on $(-\infty, -1)$ and falls on $(-1, \infty)$. At $x = -1$ there is a maximum. Since $y'' < 0$, the curve is concave down for all x.

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11. $y = x^3 - 3x + 3$

Solution:

When $y = x^3 - 3x + 3$, then $y' = 3x^2 - 3 = 3(x-1)(x+1)$ and $y'' = 6x$. The curve rises on $(-\infty, -1) \cup (1, \infty)$ and falls on $(-1, 1)$. At $x = -1$ there is a local maximum and at $x = 1$ a local minimum. The curve is concave down on $(-\infty, 0)$ and concave up on $(0, \infty)$. There is a point of inflection at $x = 0$.

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12. $y = x(6-2x)^2$

Solution:

When $y = x(6-2x)^2$, then $y' = -4x(6-2x) + (6-2x)^2 = 12(3-x)(1-x)$ and $y'' = -12(3-x) - 12(1-x) = 24(x-2)$. The curve rises on $(-\infty, 1) \cup (3, \infty)$ and falls on $(1, 3)$. The curve is concave down on $(-\infty, 2)$ and concave up $(2, \infty)$. At $x = 2$ there is a point of inflection.

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13. $y = -2x^3 + 6x^2 - 3$

Solution:

When $y = -2x^3 + 6x^2 - 3$, then $y' = -6x^2 + 12x = -6x(x-2)$ and $y'' = -12x + 12 = -12(x-1)$. The curve rises on $(0, 2)$ and falls on $(-\infty, 0)$ and $(2, \infty)$. At $x = 0$ there is a local minimum and at $x = 2$ a local maximum. The curve is concave up on $(-\infty, 1)$ and concave down on $(1, \infty)$. At $x = 1$ there is a point of inflection.

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14. $y = 1 - 9x - 6x^2 - x^3$
Solution:
When $y = 1 - 9x - 6x^2 - x^3$, then $y' = -9 - 12x - 3x^2 = -3\,(x+3)\,(x+1)$ and $y'' = -12 - 6x = -6\,(x+2)$. The curve rises on $(-3, -1)$ and falls on $(-\infty, -3)$ and $(-1, \infty)$. At $x = -1$ there is a local maximum and at $x = -3$ a local minimum. The curve is cocave up on $(-\infty, -2)$ and concave down on $(-2, \infty)$. At $x = -2$ there is a point of inflection.

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15. $y = (x-2)^3 + 1$
Solution:
When $y = (x-2)^3 + 1$, then $y' = 3\,(x-2)^2$ and $y'' = 6\,(x-2)$. The curve never falls and there are no local extrema. The curve is concave down on $(-\infty, 2)$ and concave up on $(2, \infty)$. At $x = 2$ there is a point of inflection.

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16. $y = 1 - (x+1)^3$
Solution:
When $y = 1 - (x+1)^3$, then $y' = -3\,(x+1)^2$ and $y'' = -6\,(x+1)$. The curve never rises and there are no local extrema. The curve is concave up on $(-\infty, -1)$ and concave down on $(-1, \infty)$. At $x = -1$ there is a point of inflection.

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17. $y = x^4 - 2x^2$
Solution:
When $y = x^4 - 2x^2$, then $y' = 4x^3 - 4x = 4x\,(x+1)\,(x-1)$ and $y'' = 12x^2 - 4 = 12\left(x + \dfrac{1}{\sqrt{3}}\right)\left(x - \dfrac{1}{\sqrt{3}}\right)$.
The curve rises on $(-1, 0)$ and $(1, \infty)$ and falls on $(0, 1)$. At $x = \pm 1$ there are local minima at $x = 0$ a local maximum. The curve is concave up on $\left(-\infty, -\dfrac{1}{\sqrt{3}}\right)$ and $\left(\dfrac{1}{\sqrt{3}}, \infty\right)$ and concave down on $\left(-\dfrac{1}{\sqrt{3}}, \dfrac{1}{\sqrt{3}}\right)$. At $x = \dfrac{\pm 1}{\sqrt{3}}$ there are points of inflection.

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18. $y = -x^4 + 6x^2 - 4$
Solution:
When $y = -x^4 + 6x^2 - 4$, then $y' = -4x^3 + 12x = -4x\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$ and $y'' = -12x^2 + 12 = -12\,(x+1)\,(x-1)$. The curve rises on $\left(-\infty, -\sqrt{3}\right)$ and $\left(0, \sqrt{3}\right)$, and falls on $\left(-\sqrt{3}, 0\right)$ and $\left(\sqrt{3}, \infty\right)$. At $x = \pm\sqrt{3}$ there are local maxima and at $x = 0$ a local minimum. The curve is concave up on $(-1, 1)$ and concave down on $(-\infty, -1)$ and $(1, \infty)$. At $x = \pm 1$ there are points of inflection.

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19. $y = 4x^3 - x^4$
Solution:

When $y = 4x^3 - x^4$, then $y' = 12x^2 - 4x^3 = 4x^2(3-x)$ and $y'' = 24x - 12x^2 = 12x(2-x)$. The curve rises on $(-\infty, 3)$ and falls on $(3, \infty)$. At $x = 3$ there is a local maximum, but there is no local minimum. The graph is concave up on $(0, 2)$ and concave down on $(-\infty, 0)$ and $(2, \infty)$. There are inflection points on $x = 0$ and $x = 2$.

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4. A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?

Solution:

The area of the rectangle is $A = 2xy = 2x\left(12 - x^2\right)$, where $0 \leq x \leq \sqrt{12}$. Solving $A'(x) = 0 \implies 24 - 6x^2 = 0 \implies x = -2$ or 2. Now -2 is not in the domain, and since $A(0) = 0$ and $A\left(\sqrt{12}\right) = 0$, we conclude that $A(2) = 32$ square units is the maximum area. The dimensions are 4 units by 8 units.

Sketch the graph of $f(x) = \dfrac{6x}{x^2+1}$.

Solution:

The function $f(x) = \dfrac{6x}{x^2+1}$ is differentiable everwhere, and its derivative $f'(x) = \dfrac{6\left(1-x^2\right)}{\left(x^2+1\right)^2}$ is zero if and only if $x = \pm 1$. Therefore f has exactly two critical points, namely $x = \pm 1$. Since $f'(x) < 0$ if $x < -1$, $f'(x) > 0$ if $-1 < x < 1$, $f'(x) < 0$ if $x > 1$, it follows from the first derivative test that f has a strict local maximum at $x = 1$, equal to $f(1) = 3$, and a strict local minimumat $x = -1$, equal to $f(1) = -3$. By the monotonicity test, f is increasing on $(-1, 1)$ and decreasing on $(-\infty, -1)$ and $(1, \infty)$. Since $f(x) > 0$ if $x > 0$ and $f(x) < 0$ if $x < 0$, this shows that the local extrema of f at $x = \pm 1$ are absolute.
