

ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science
MATH 156 Calculus for Engineering II
Practice Problems-c
Final Exam
May 26, 2008
9:00

1. 15.4 TRIPLE INTEGRALS IN RECTANGULAR COORDINATES

(p. 1107) Find the volumes of the regions in Exercises 23-26.

23. The region between the cylinder $z = y^2$ and the xy -plane that is bounded by the planes $x = 0, x = 1, y = -1, y = 1$.

Solution:

$$V = \int_0^1 \int_{-1}^1 \int_0^{y^2} dz \, dy \, dx = \cdots = \frac{2}{3}$$

24. The region in the first octant bounded by the coordinate planes and the planes $x + z = 1, y + 2z = 2$.

Solution:

$$V = \int_0^1 \int_0^{1-x} \int_0^{2-2x} dy \, dz \, dx = \cdots = \frac{2}{3}$$

25. The region in the first octant bounded by the coordinate planes and the planes $y + z = 2$ and the cylinder $x = 4 - y^2$.

Solution:

$$V = \int_0^4 \int_0^{\sqrt{4-x}} \int_0^{2-y} dz \, dy \, dx = \cdots = \frac{20}{3}$$

26. The wedge cut from the cylinder $x^2 + y^2 = 1$ by the planes $z = -y$ and $z = 0$.

Solution:

$$V = 2 \int_0^1 \int_{-\sqrt{1-x^2}}^0 \int_0^{-y} dz \, dy \, dx = \cdots = \frac{2}{3}$$

27. The tetrahedron in the first octant bounded by the coordinate planes and the plane passing through $(1, 0, 0), (0, 2, 0), (0, 0, 3)$.

Solution:

$$\int_0^1 \int_0^{2-2x} \int_0^{3-3x-3y/2} dz \, dy \, dx = \cdots = 1$$

28. The region in the first octant bounded by the coordinate planes, the plane $y = 1 - x$, and the surface $z = \cos(\pi x/2)$, $0 \leq x \leq 1$

Solution: $V = \int_0^1 \int_0^{1-x} \int_0^{\cos(\pi x/2)} dz \, dy \, dx = \cdots = \frac{4}{\pi^2}.$

29. The region common to the interiors of the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.

Solution: $V = 8 \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2}} dz \, dy \, dx = \cdots = \frac{16}{3}.$

30. The region in the first octant bounded by the coordinate planes and the surface $z = 4 - x^2 - y$.

Solution: $V = \int_0^2 \int_0^{4-x^2} \int_0^{4-x^2-y} dz \, dy \, dx = \cdots = \frac{128}{15}.$

31. The region in the first octant bounded by the coordinate planes, the plane $x + y = 4$, and the cylinder $y^2 + 4z^2 = 16$.

Solution: $V = \int_0^4 \int_0^{(\sqrt{16-y^2})/2} \int_0^{4-y} dx \, dz \, dy = \cdots = \frac{32}{3}.$

32. The region cut from the cylinder $x^2 + y^2 = 4$ by the plane $x + z = 3$.

Solution: $V = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{3-x} dz \, dy \, dx = \cdots = 12\pi.$

33. The region between the planes $x + y + 2z = 2$ and $2x + 2y + z = 4$ in the first octant.

Solution: $V = \int_0^2 \int_0^{2-x} \int_{(2-x-y)/2}^{4-2x-y} dz \, dy \, dx = \cdots = 2.$

34. The finite region bounded by the planes $z = x$, $x + z = 8$, $z = y$, $y = 8$, and $z = 0$.

Solution: $V = \int_0^4 \int_z^8 \int_z^{8-z} dx \, dy \, dz = \cdots = \frac{320}{3}.$

35. The region cut from the elliptical cylinder $x^2 + 4y^2 \leq 4$ by the xy -plane and the plane $z = x + 2$.

Solution: $V = 2 \int_{-2}^2 \int_0^{\sqrt{4-x^2}/2} \int_0^{x+2} dz \, dy \, dx = \cdots = 4\pi.$

36. The region bounded in the back by the plane $x = 0$, on the front and sides by the parabolic cylinder $x = 1 - y^2$, on the top by the paraboloid $z = x^2 + y^2$, and on the bottom by the xy -plane.

Solution: $V = 2 \int_0^1 \int_0^{1-y^2} \int_0^{x^2+y^2} dz \, dx \, dy = \cdots = \frac{4}{7}.$

2. CHANGING THE ORDER OF INTEGRATION

Evaluate the integrals in Exercises 41-44 by changing the order of integration in an appropriate way.

41. $\int_0^4 \int_0^1 \int_{2y}^2 \frac{4 \cos(x^2)}{2\sqrt{z}} dx dy dz.$

Solution:

$$\begin{aligned} \int_0^4 \int_0^1 \int_{2y}^2 \frac{4 \cos(x^2)}{2\sqrt{z}} dx dy dz &= \int_0^4 \int_0^2 \int_0^{x/2} \frac{4 \cos(x^2)}{2\sqrt{z}} dy dx dz = \int_0^4 \int_0^2 \frac{x \cos(x^2)}{\sqrt{z}} dx dz = \\ &= \int_0^4 \left(\frac{\sin 4}{2} \right) z^{-1/2} dz \\ &= [(\sin 4) z^{1/2}]_0^4 = 2 \sin 4 \end{aligned}$$

42. $\int_0^1 \int_0^1 \int_{x^2}^1 12xz e^{zy^2} dy dx dz.$

Solution:

$$\begin{aligned} \int_0^1 \int_0^1 \int_{x^2}^1 12xz e^{zy^2} dy dx dz &= \int_0^1 \int_0^1 \int_0^{\sqrt{y}} 12xz e^{zy^2} dx dy dz \\ \int_0^1 \int_0^1 6yz e^{zy^2} dy dz &= \int_0^1 [3e^{zy^2}] dy dz = \cdots = 3e - 6 \end{aligned}$$

43. $\int_0^1 \int_{\sqrt[3]{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin \pi y^2}{y^2} dx dy dz.$

Solution:

$$\begin{aligned} \int_0^1 \int_{\sqrt[3]{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin \pi y^2}{y^2} dx dy dz &= \int_0^1 \int_{\sqrt[3]{z}}^1 \frac{4\pi \sin(\pi y^2)}{y^2} dy dz = \int_0^1 \int_0^{y^2} \frac{4\pi \sin(\pi y^2)}{y^2} dz dy \\ \int_0^1 4\pi y \sin(\pi y^2) dy &= [-2 \cos(\pi y^2)]_0^1 = \cdots = 4 \end{aligned}$$

44. $\int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin 2z}{4-z} dy dz dx$

Solution:

$$\begin{aligned} \int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin 2z}{4-z} dy dz dx &= \int_0^2 \int_0^{4-x^2} \frac{x \sin 2z}{4-z} dz dx = \int_0^4 \int_0^{\sqrt{4-z}} \left(\frac{\sin 2z}{4-z} \right) x dx dz \\ &= \int_0^4 \left(\frac{\sin 2z}{4-z} \right) \frac{1}{2} (4-z) dz = \left[-\frac{1}{4} \cos 2z \right]_0^4 = \left[-\frac{1}{4} + \frac{1}{2} \sin^2 z \right]_0^4 = \frac{\sin^2 4}{2} \end{aligned}$$
