ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MATH 156 Calculus for Engineering II

Practice Problems-d

Final Exam

May 26, 2008

9:00

1. 15.4 Triple Integrals in Rectangular Coordinates

(p. 1107) Find the volumes of the regions in Exercises 23-26.

23. The region between the cylinder $z = y^2$ and the xy-plane that is bounded by the planes $x = 0, x = 1, y = -1, y = 1$.

Solution:

$$V = \int_0^1 \int_{-1}^1 \int_0^{y^2} dz\ dy\ dx = \cdots = \frac{2}{3}$$

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24. The region in the first octant bounded by the coordinate planes and the planes $x + z = 1, y + 2z = 2$.

Solution:

$$V = \int_0^1 \int_0^{1-x} \int_0^{2-2x} dy\ dz\ dx = \cdots = \frac{2}{3}$$

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25. The region in the first octant bounded by the coordinate planes and the planes $y + z = 2$ and the cylinder $x = 4 - y^2$.

Solution:

$$V = \int_0^4 \int_0^{\sqrt{4-x}} \int_0^{2-y} dz\ dy\ dx = \cdots = \frac{20}{3}$$

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26. The wedge cut from the cylinder $x^2 + y^2 = 1$ by the planes $z = -y$ and $z = 0$.

Solution:

$$V = 2\int_0^1 \int_{-\sqrt{1-x^2}}^0 \int_0^{-y} dz\ dy\ dx = \cdots = \frac{2}{3}$$

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27. The tetrahedron in the first octant bounded by the coordinate planes and the plane passing through $(1, 0, 0), (0, 2, 0), (0, 0, 3)$.

Solution:

$$\int_0^1 \int_0^{2-2x} \int_0^{3-3x-3y/2} dz\ dy\ dx = \cdots = 1$$

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28. The region in the first octant bounded by the coordinate planes, the plane $y = 1 - x$, and the surface $z = \cos(\pi x/2), 0 \le x \le 1$

Solution: $V = \int_0^1 \int_0^{1-x} \int_0^{\cos(\pi x/2)} dz\ dy\ dx = \cdots = \frac{4}{\pi^2}.$

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29. The region common to the in teriors of the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.

Solution: $V = 8 \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2}} dz\ dy\ dx = \cdots = \frac{16}{3}.$

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30. The region in the first octant bounded by the coordinate planes and the surface $z = 4 - x^2 - y$.

Solution: $V = \int_0^2 \int_0^{4-x^2} \int_0^{4-x^2-y} dz\ dy\ dx = \cdots = \frac{128}{15}.$

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31. The region in the first octant bounded by the coordinate planes, the plane $x + y = 4$, and the cyliner $y^2 + 4z^2 = 16$.

Solution: $V = \int_0^4 \int_0^{\left(\sqrt{16-y^2}\right)/2} \int_0^{4-y} dx\ dz\ dy = \cdots = \frac{32}{3}.$

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32. The region cut from the cylinder $x^2 + y^2 = 4$ by the plane $x + z = 3$.

Solution: $V = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{3-x} dz\ dy\ dx = \cdots = 12\pi.$

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33. The region between the planes $x + y + 2z = 2$ and $2x + 2y + z = 4$ in the first octant.

Solution: $V = \int_0^2 \int_0^{2-x} \int_{(2-x-y)/2}^{4-2x-y} dz\ dy\ dx = \cdots = 2.$

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34. The finite region bounded by the planes $z = x, x + z = 8, z = y, y = 8$, and $z = 0$.

Solution: $V = \int_0^4 \int_z^8 \int_z^{8-z} dx\ dy\ dz = \cdots = \frac{320}{3}.$

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35. The region cut from the elliptical cylinder $x^2 + 4y^2 \le 4$ by the xy-plane and the plane $z = x + 2$.

Solution: $V = 2 \int_{-2}^2 \int_0^{\sqrt{4-x^2}/2} \int_0^{x+2} dz\ dy\ dx = \cdots = 4\pi.$

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36. The region bounded in the back by the plane $x = 0$, on the front and sides by the parabolic cylinder $x = 1 - y^2$, on the top by the paraboloid $z = x^2 + y^2$, and on the bottom by the xy-plane.

Solution: $V = 2 \int_0^1 \int_0^{1-y^2} \int_0^{x^2+y^2} dz\ dx\ dy = \cdots = \frac{4}{7}.$

2. Changing the order of integration

Evaluate the integrals in Exercises 41-44 by changing the order of integration in an appropriate way.

41. $\int_0^4 \int_0^1 \int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\ dx\ dy\ dz.$

Solution:

$$\int_0^4 \int_0^1 \int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{z}}\ dx\ dy\ dz = \int_0^4 \int_0^2 \int_0^{x/2} \frac{4\cos(x^2)}{2\sqrt{z}}\ dy\ dx\ dz = \int_0^4 \int_0^2 \frac{x\cos(x^2)}{\sqrt{z}}\ \ dx\ dz =$$

$$\int_0^4 \left(\frac{\sin 4}{2}\right) z^{-1/2}\ dz$$

$$= \left[(\sin 4)\, z^{1/2}\right]_0^4 = 2\sin 4$$

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42. $\int_0^1 \int_0^1 \int_{x^2}^1 12xze^{zy^2}\ dy\ dx\ dz.$

Solution:

$$\int_0^1 \int_0^1 \int_{x^2}^1 12xze^{zy^2}\ dy\ dx\ dz = \int_0^1 \int_0^1 \int_0^{\sqrt{y}} 12xze^{zy^2}\ dx\ dy\ dz$$

$$\int_0^1 \int_0^1 6yze^{zy^2}\ dy\ dz = \int_0^1 \left[3e^{zy^2}\right]\ dy\ dz = \cdots = 3e - 6$$

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43. $\int_0^1 \int_{\sqrt[3]{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x}\sin \pi y^2}{y^2}\ dx\ dy\ dz.$

Solution:

$$\int_0^1 \int_{\sqrt[3]{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x}\sin \pi y^2}{y^2}\ dx\ dy\ dz = \int_0^1 \int_{\sqrt[3]{z}}^1 \frac{4\pi\sin(\pi y^2)}{y^2}\ dy\ dz = .\int_0^1 \int_0^{y^2} \frac{4\pi\sin\left(\pi y^2\right)}{y^2}\ dz\ dy$$

$$.\int_0^1 4\pi y\sin(\pi y^2)\ \ dy = \left[-2\cos(\pi y^2)\right]_0^1 = \cdots = 4$$

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44. $\int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin 2z}{4-z}\ dy\ dz\ dx$

Solution:

$$\int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin 2z}{4-z}\ dy\ dz\ dx = \int_0^2 \int_0^{4-x^2} \frac{x\sin 2z}{4-z}\ \ dz\ dx = \int_0^4 \int_0^{\sqrt{4-z}} \left(\frac{\sin 2z}{4-z}\right) x\ dx\ dz$$

$$= \int_0^4 \left(\frac{\sin 2z}{4-z}\right) \frac{1}{2}(4-z)\ \ dz = \left[-\frac{1}{4}\cos 2z\right]_0^4 = \left[-\frac{1}{4} + \frac{1}{2}\sin^2 z\right]_0^4 = \frac{\sin^2 4}{2}$$
