

ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science
MATH 156 Calculus for Engineering II
Practice Problems-d
Final Exam
May 26, 2008
9:00

1. 15.6 TRIPLE INTEGRALS IN CYLINDRICAL AND SPHERICAL COORDINATES

(p. 1127)

49. Find the volume of the portion of the solid sphere $\rho \leq a$ that lies between the cones $\phi = \pi/3$ and $\phi = 2\pi/3$.

Solution:

$$V = \int_0^{2\pi} \int_{\pi/3}^{2\pi/3} \int_0^a \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \dots = \frac{2\pi a^3}{3}$$

50. Find the volume of the region cut from the solid sphere $\rho \leq a$ by the half-planes $\theta = 0$ and $\theta = \pi/6$ in the first octant.

Solution:

$$V = \int_0^{\pi/6} \int_0^{\pi/2} \int_0^a \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \dots = \frac{a^3 \pi}{18}$$

51. Find the volume of the smaller region cut from the solid sphere $\rho \leq 2$ by the plane $z = 1$.

Solution:

$$V = \int_0^{2\pi} \int_0^{\pi/3} \int_{\sec \phi}^2 \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \dots = \frac{5\pi}{3}$$

52. Find the volume of the solid enclosed by the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 2$.

Solution:

$$\begin{aligned} V &= 4 \int_0^{\pi/2} \int_0^{\pi/4} \int_{\sec \phi}^{2 \sec \phi} \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta = \frac{4}{3} \int_0^{\pi/2} \int_0^{\pi/4} (8 \sec^3 \phi - \sec^3 \phi) \sin \phi \, d\phi \, d\theta \\ &= \frac{28}{3} \int_0^{\pi/2} \int_0^{\pi/4} \sec^3 \phi \sin \phi \, d\phi \, d\theta = \frac{28}{3} \int_0^{\pi/2} \int_0^{\pi/4} \tan \phi \sec^2 \phi \, d\phi \, d\theta \\ &= \frac{28}{3} \int_0^{\pi/2} \left[\frac{1}{2} \tan^2 \phi \right]_0^{\pi/4} d\theta = \dots = \frac{7\pi}{3} \end{aligned}$$

53. Find the volume of the region bounded below by the plane $z = 0$ laterally by the cylinder $x^2 + y^2 = 1$, and above by the paraboloid $z = x^2 + y^2$.

Solution:

$$V = 4 \int_0^{\pi/2} \int_0^1 \int_0^{r^2} dz \, r \, dr \, d\theta = \cdots = \frac{\pi}{2}$$

54. Find the volume of the region bounded below by the paraboloid $z = x^2 + y^2$, laterally by the cylinder $x^2 + y^2 = 1$, and above by the paraboloid $z = x^2 + y^2 + 1$.

Solution:

$$V = 4 \int_0^{\pi/2} \int_0^1 \int_{r^2}^{r^2+1} dz \, r \, dr \, d\theta = \cdots = \pi.$$

55. Find the volume of the solid The region common to the interiors of the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.

$$\text{Solution: } V = 8 \int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2}} dz \, dy \, dx = \cdots = \frac{16}{3}.$$

30. The region in the first octant bounded by the coordinate planes and the surface $z = 4 - x^2 - y$.

$$\text{Solution: } V = \int_0^2 \int_0^{4-x^2} \int_0^{4-x^2-y} dz \, dy \, dx = \cdots = \frac{128}{15}.$$

31. The region in the first octant bounded by the coordinate planes, the plane $x + y = 4$, and the cylinder $y^2 + 4z^2 = 16$.

$$\text{Solution: } V = \int_0^4 \int_0^{(\sqrt{16-y^2})/2} \int_0^{4-y} dx \, dz \, dy = \cdots = \frac{32}{3}.$$

32. The region cut from the cylinder $x^2 + y^2 = 4$ by the plane $x + z = 3$.

$$\text{Solution: } V = \int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^{3-x} dz \, dy \, dx = \cdots = 12\pi.$$

33. The region between the planes $x + y + 2z = 2$ and $2x + 2y + z = 4$ in the first octant.

$$\text{Solution: } V = \int_0^2 \int_0^{2-x} \int_{(2-x-y)/2}^{4-2x-y} dz \, dy \, dx = \cdots = 2.$$

34. The finite region bounded by the planes $z = x, x + z = 8, z = y, y = 8$, and $z = 0$.

$$\text{Solution: } V = \int_0^4 \int_z^8 \int_z^{8-z} dx \, dy \, dz = \cdots = \frac{320}{3}.$$

35. The region cut from the elliptical cylinder $x^2 + 4y^2 \leq 4$ by the xy -plane and the plane $z = x + 2$.

$$\text{Solution: } V = 2 \int_{-2}^2 \int_0^{\sqrt{4-x^2}/2} \int_0^{x+2} dz \, dy \, dx = \cdots = 4\pi.$$

36. The region bounded in the back by the plane $x = 0$, on the front and sides by the parabolic cylinder $x = 1 - y^2$, on the top by the paraboloid $z = x^2 + y^2$, and on the bottom by the xy -plane.

Solution: $V = 2 \int_0^1 \int_0^{1-y^2} \int_0^{x^2+y^2} dz \, dx \, dy = \cdots = \frac{4}{7}.$

2. CHANGING THE ORDER OF INTEGRATION

Evaluate the integrals in Exercises 41-44 by changing the order of integration in an appropriate way.

41. $\int_0^4 \int_0^1 \int_{2y}^2 \frac{4 \cos(x^2)}{2\sqrt{z}} \, dx \, dy \, dz.$

Solution:

$$\begin{aligned} \int_0^4 \int_0^1 \int_{2y}^2 \frac{4 \cos(x^2)}{2\sqrt{z}} \, dx \, dy \, dz &= \int_0^4 \int_0^2 \int_0^{x/2} \frac{4 \cos(x^2)}{2\sqrt{z}} \, dy \, dx \, dz = \int_0^4 \int_0^2 \frac{x \cos(x^2)}{\sqrt{z}} \, dx \, dz = \\ &= \int_0^4 \left(\frac{\sin 4}{2} \right) z^{-1/2} \, dz \\ &= [(\sin 4) z^{1/2}]_0^4 = 2 \sin 4 \end{aligned}$$

42. $\int_0^1 \int_0^1 \int_{x^2}^1 12xz e^{zy^2} \, dy \, dx \, dz.$

Solution:

$$\begin{aligned} \int_0^1 \int_0^1 \int_{x^2}^1 12xz e^{zy^2} \, dy \, dx \, dz &= \int_0^1 \int_0^1 \int_0^{\sqrt{y}} 12xz e^{zy^2} \, dx \, dy \, dz \\ &= \int_0^1 \int_0^1 6yz e^{zy^2} \, dy \, dz = \int_0^1 [3e^{zy^2}] \, dy \, dz = \cdots = 3e - 6 \end{aligned}$$

43. $\int_0^1 \int_{\sqrt[3]{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin \pi y^2}{y^2} \, dx \, dy \, dz.$

Solution:

$$\begin{aligned} \int_0^1 \int_{\sqrt[3]{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin \pi y^2}{y^2} \, dx \, dy \, dz &= \int_0^1 \int_{\sqrt[3]{z}}^1 \frac{4\pi \sin(\pi y^2)}{y^2} \, dy \, dz = \int_0^1 \int_0^{y^2} \frac{4\pi \sin(\pi y^2)}{y^2} \, dz \, dy \\ &= \int_0^1 4\pi y \sin(\pi y^2) \, dy = [-2 \cos(\pi y^2)]_0^1 = \cdots = 4 \end{aligned}$$

44. $\int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin 2z}{4-z} \, dy \, dz \, dx$

Solution:

$$\begin{aligned} \int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin 2z}{4-z} \, dy \, dz \, dx &= \int_0^2 \int_0^{4-x^2} \frac{x \sin 2z}{4-z} \, dz \, dx = \int_0^4 \int_0^{\sqrt{4-z}} \left(\frac{\sin 2z}{4-z} \right) x \, dx \, dz \\ &= \int_0^4 \left(\frac{\sin 2z}{4-z} \right) \frac{1}{2} (4-z) \, dz = \left[-\frac{1}{4} \cos 2z \right]_0^4 = \left[-\frac{1}{4} + \frac{1}{2} \sin^2 z \right]_0^4 = \frac{\sin^2 4}{2} \end{aligned}$$
