ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MATH 156 Calculus for Engineering II

Practice Problems

1^{st} Midterm

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1. Convergent Series

(p.840) Find the sum of the series in Exercises 19-24.

19. $\displaystyle\sum_{n=3}^{\infty} \frac{1}{(2n-3)(2n-1)}$

20. $\displaystyle\sum_{n=2}^{\infty} \frac{-2}{n(n+1)}$

21. $\displaystyle\sum_{n=1}^{\infty} \frac{9}{(3n-1)(3n+21)}$

22. $\displaystyle\sum_{n=3}^{\infty} \frac{-8}{(4n-3)(4n+1)}$

23. $\displaystyle\sum_{n=0}^{\infty} e^{-n}$

24. $\displaystyle\sum_{n=3}^{\infty} \frac{1}{(2n-3)(2n-1)}$

25. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3}{4^n}$

2. Convergent or Divergent Series

Which of the series in Exercises 25-40 converge absolutely, which converge conditionally, and which diverge? Give reasons for your answers.

25. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

26. $\displaystyle\sum_{n=3}^{\infty} \frac{-5}{n}$

27. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

28. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n^3}$

29. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

30. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\,(\ln n)^2}$

31. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

32. $\displaystyle\sum_{n=3}^{\infty} \frac{\ln n}{\ln(\ln n)}$

33. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n^2+1}}$

34. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\,3n^2}{n^3+1}$

35. $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n!}$

36. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\,(n^2+1)}{2n^2+n-1}$

37. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{n!}$

38. $\displaystyle\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$

39. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n\,(n+1)\,(n+2)}}$

40. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$

3. Power Series

In Exercises 41-50, (a) find the series' radius and interval of convergence. Then identify the values of x for which the series converges (b) absolutely and (c) conditionally.

41. $\displaystyle\sum_{n=1}^{\infty} \frac{(x+4)^n}{n3^n}$

42. $\displaystyle\sum_{n=1}^{\infty} \frac{(x-1)^{2n-2}}{(2n-1)!}$

43. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\,(3x-1)^n}{n^2}$

44. $\displaystyle\sum_{n=0}^{\infty} \frac{(n+1)\,(2x+1)^n}{(2n+1)\,2^n}$

45. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^n}$

46. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

47. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)\,x^{2n-1}}{3^n}$

48. $\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{2n+1}}{2n+1}$

49. $\sum_{n=1}^{\infty} (\csc hn)\, x^n$

50. $\sum_{n=1}^{\infty} (\coth n)\, x^n$

4. Maclaurin Series

Each of the series in Exercises 51-56 is the value of the Taylor series at $x = 0$ of a function $f(x)$ at a particular point. What function and what point? What is the sum of the series?

51. $1 - \frac{1}{4} + \frac{1}{16} - \cdots + (-1)^n \frac{1}{4^n} + \cdots$

52. $\frac{2}{3} - \frac{4}{18} + \frac{8}{81} - \cdots + (-1)^{n-1} \frac{2^n}{n3^n} + \cdots$

53. $\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \cdots + (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} + \cdots$

54. $1 - \frac{\pi^2}{9.2!} + \frac{\pi^4}{81.4!} - \cdots + (-1)^n \frac{\pi^{2n}}{3^{2n}(2n)!} + \cdots$

55. $1 + \ln 2 + \frac{(\ln 2)^2}{2!} + \cdots + \frac{(\ln 2)^n}{n!} + \cdots$

56. $\frac{1}{\sqrt{3}} - \frac{1}{9\sqrt{3}} + \frac{1}{45\sqrt{3}} - \cdots + (-1)^{n-1} \frac{1}{(2n-1)\left(\sqrt{3}\right)^{2n-1}} + \cdots$

Find the Taylor series at $x = 0$ for the functions in Exercises 57-64.

57. $\frac{1}{1-2x}$

58. $\frac{1}{1+x^3}$

59. $\sin \pi x$

60. $\sin \frac{2x}{3}$

61. $\cos\left(x^{5/2}\right)$

62. $\cos \sqrt{5x}$

63. $e^{(\pi x/2)}$

64. e^{-x^2}

5. Taylor Series

In Exercises 65-68, find the first four nonzero terms of the Taylor series generated by f at $x = a$.

65. $f(x) = \sqrt{3 + x^2}$ at $x = -1$

66. $f(x) = \frac{1}{1-x}$ at $x = 2$

67. $f(x) = \frac{1}{x+1}$ at $x = 3$

68. $f(x) = \frac{1}{x}$ at $x = a > 0$

6. Nonelementary Integrals

Use series to approximate the values of the integrals in Exercises 77-80 with an error of magnitude less than 10^{-8}. The answer section gives the integrals' values rounded to 10 decimal places.)

77. $\int_0^{1/2} e^{-x^2}\,dx$

78. $\int_0^1 x\sin\left(x^3\right)dx$

79. $\int_0^{1/2} \frac{\tan^{-1}x}{x}dx$

80. $\int_0^{1/64} \frac{\tan^{-1}x}{\sqrt{x}}dx$

7. Indeterminate Forms

In Exercises 81-86 use power series to evaluate the limit.

81. $\lim_{x\to 0} \frac{7\sin x}{e^{2x}-1}$

82. $\lim_{\theta\to 0} \frac{e^{\theta}-e^{-\theta}-2\theta}{\theta-\sin\theta}$

83. $\lim_{t\to 0} \left(\frac{1}{2-2\cos t}-\frac{1}{t^2}\right)$

84. $\lim_{h\to 0} \frac{(\sinh)/h-\cosh}{h^2}$

85. $\lim_{z\to 0} \frac{1-\cos^2 z}{\ln(1-z)+\sin z}$

86. $\lim_{y\to 0} \frac{y^2}{\cos y-\cosh y}$

87. Use a series representation of $\sin 3x$ to find values of r and s for which

$$\lim_{x\to 0}\left(\frac{\sin 3x}{x^3}+\frac{r}{x^2}+s\right)=0.$$