ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MATH 156 Calculus for Engineering II

Practice Problems

2^{nd} Midterm

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1. Lines and Planes in Space

(p. 887) Find parametric equations for the lines in Exercises 1-12.

1. The line through the point $\mathbf{i}+\mathbf{j}+\mathbf{k}$. parallel to the vector $\mathbf{i}+\mathbf{j}+\mathbf{k}$.

Solution: the direction $\mathbf{i}+\mathbf{j}+\mathbf{k}$. and $\mathbf{i}+\mathbf{j}+\mathbf{k}$.$\Longrightarrow x = 3+t, y = -4+t, z = -1+t$.

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2. The line through P(1,2,-1)andQ(-1,0,1)

Solution:the direction $\overrightarrow{PQ} = -2i-2j+2k$ and $P(1,2-1) \Longrightarrow x = 1-2t, y = 2-2t, z = -1+2t$

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3. The line through $P(-2,0,3)$ and $Q(3,5,-2)$

Solution:the direction $\overrightarrow{PQ} = 5i+5j-5k$ and $P(-2,0,3) \Longrightarrow x = -2+5t, y = 5t, z = -1$

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4. The line through $P(1,2,0)$ and $Q(1,1,-1)$

Solution::the direction $\overleftrightarrow{PQ} = -j-k$ and $P(1,2,0) \Longrightarrow x = 1, y = 2-t, z = -t$

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5. The line through the origin parellel to the vector $2j+k$

Solution: the direction $2j+k$ and $P(0,0,0) \Longrightarrow x = 0, y = 2t, z = t$

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6. The line through the point $(3,-2,1)$ parallel to the line $x = 1+2t, y = 2-t, z = 3t$

Solution: the direction $2i-j+3k$ and $P(3,-2,1) \Longrightarrow x = 3+2t, y = -2, z = 1+3t$

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7. The line through $(1,1,1)$ parallel to the z-axis

Solution: the direction k and $P(1,1,1) \Longrightarrow x = 1, y = 1, z = 1+t$

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8. The line through $(2, 4, 5)$ perpendicular to the plane $3x + 7y - 5z = 21$
Solution: the direction $3i + 7j - 5k$ and $P(2, 4, 5) \Longrightarrow x = 2 + 3t, y = 4 + 7t, z = 5 - 5t$

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9. The line through $(0, -7, 0)$ perpendicular to the plane $x + 2y + 2z = 13$
Solution: the direction $i + 2j + 2k$ and $P(0, -7, 0) \Longrightarrow x = t, y = -7 + 2t, z = 2t$

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10. The line through $(2, 3, 0)$ perpendicular to the vectors $u = i + 2j + 3k$ and $v = 3i + 4j + 5k$
Solution:the direction is $A \times B = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & 4 & 5 \end{vmatrix} = -2i + 4j - 2k$ and $P(2, 3, 0) \Longrightarrow x = 2 - 2t, y = 3 + 4t, z = -2t$

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11. The $\times$-axis
Solution: the direction i and $P(0, 0, 0) \Longrightarrow x = 0, y = 0, z = 0$

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12. The z-axis.
Solution: the direction k and $P(0, 0, 0) \Longrightarrow x = 0, y = 0, z = t$

Find parametrizations for the line segment joining the points in exercises 13-20. draw coordinate axes and sketch eaach segment indicating the directions of increaing t for your parametrization

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13. $(0, 0, 0), (1, 1, 3/2)$.
Solution: the direction $\overrightarrow{PQ} = i + j + \frac{3}{2}k$ and $P(0, 0, 0) \Longrightarrow x = t$
$y = t, z = \frac{3}{2}t$ where $0 \leq t \leq 1$

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14. $(0, 0, 0), (1, 0, 0)$
Solution: the direction $\overrightarrow{PQ} = i$ and $P(0, 0, 0) \Longrightarrow x = t, y = 0, z = 0$
where $0 \leq t \leq 1$

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15. $(1, 0, 0), (1, 1, 0)$.
Solution: the direction $\overrightarrow{PQ} = j$ and $P(1, 1, 0) \Longrightarrow x = 1, y = 1 + t, z =$, where $-1 \leq t \leq 0$

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16. $(1, 1, 0), (1, 1, 1)$
Solution: the direction $\overrightarrow{PQ} = k$ and $P(1, 1, 0) \Longrightarrow x = 1, y = 1, z = t$

where $0 \le t \le 1$

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17. $(0,1,1),(0,-1,1)$
Solution: the direction $\overrightarrow{PQ} = -2j$ and $P(0,1,1) \Longrightarrow x = 0, y = 1-2t, z = 1$ where $0 \le t \le 1$

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18. $(0,2,0),(3,0,0)$
Solution: the direction $\overrightarrow{PQ} = 3i - 2j$ and $P(0,2,0) \Longrightarrow x = 3t$
$y = 2-2t, z = 0$ where $0 \le t \le 1$

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19. $(2,0,2),(0,2,0)$
Solution: the direction $\overrightarrow{PQ} = -2i + 2j - 2k$ and $P(2,0,2) \Longrightarrow x = 2-2t, y = 2t, y = 2-2t$
where $0 \le t \le 1$

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20. $(1,0,-1),(0,3,0)$
Solution: the direction $\overrightarrow{PQ} = -i+3j+k$ and $P(1,0,-1) \Longrightarrow x = 1-t, y = 3t, z = -1+t$,where,
$0 \le t \le 1$

2. PLANES

find equations for the planes in exercises

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21. The plane through $P_0(0,2,-1)$ normal to $n = 3i - 2j - k$
Solution:$3(x-0) + (-2)(y-2) + (-1)(z+1) = 0 \Longrightarrow 3x - 2y - z = -3$

22.The plane through $(1,-1,3)$ parallel to the plane
$3x + y + z = 7$
Solution:
$3(x-1) + (1)(y+1) + (1)(z-3) = 0 \Longrightarrow 3x + y + z = 5$

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23. The plane through $(1,1,-1)$,and$(2,0,2), and (0,-2,1)$
Solution:$\overrightarrow{PQ} = i - j + 3k, \overrightarrow{PS} = -i - 3j + 2k \Longrightarrow \overrightarrow{PQ} x \overrightarrow{PS} = \begin{vmatrix} i & j & k \\ 1 & -1 & 3 \\ -1 & -3 & 2 \end{vmatrix} = 7i - 5j - 4k$ is
normal to the plane
$\Longrightarrow 7(x-2) + (-5)(y-0) + (-4)(z-2) = 0 \Longrightarrow 7x - 5y - 4z = 6$

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24. The plane through $(2,4,5)$,$(1,5,7)$,and $(-1,6,8)$
Solution:
$\overrightarrow{PQ}=-\mathbf{i}+\mathbf{j}+2\mathbf{k}, \overrightarrow{PS}=-3\mathbf{i}+2\mathbf{j}+3\mathbf{k} \Longrightarrow \overrightarrow{PQ}x\overrightarrow{PS}=\begin{vmatrix} i & j & k \\ -1 & 1 & 2 \\ -3 & 2 & 3 \end{vmatrix}=-i-3j+k$ is normal to the plane
$\Longrightarrow (-1)(x-1)+(-3)(y-5)+(1)(z-7)=0 \Longrightarrow x+3y-z=9$

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25. The plane through $P_0\,(2,4,5)$ perpendicular to the line

$$x=5+t, y=1+3t, z=4t$$

Solution:
$\mathbf{n}=\mathbf{i}+3\mathbf{j}+4\mathbf{k},\ P\,(2,4,5)=(1)\,(x-2)+(3)\,(y-4)+(4)\,(z-5)=0 \Longrightarrow x+3y+4z=34$

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26. The plane through $A\,(1,-2,1)$ perpendicular to the vector from the origin to A
Solution:
$\mathbf{n}=\mathbf{i}-2\mathbf{j}+\mathbf{k},\ P\,(1,-2,1)=(1)\,(x-1)+(-2)\,(y+2)+(1)\,(z-1)=0 \Longrightarrow x-2y+z=6$

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27. Find the of intersection of the lines $x=2t+1, y=3t+2, z=4t+3$, and $x=s+2, y=2s+4, z=-4s-1$,and then find the plane determined by these lines.
Solution:
$\begin{cases} x=2t+1=s+2 \\ y=3t+2=2s+4 \end{cases} \Longrightarrow \begin{cases} 2t-s=1 \\ 3t-2s=2 \end{cases} \Longrightarrow \begin{cases} 4t-2s=2 \\ 3t-2s=2 \end{cases} \Longrightarrow t=0, s=-1$; then

$z=4t+3=-4s-1 \Longrightarrow 4\,(0)+3=(-4)\,(-1)-1$ is satisfied $\Longrightarrow$ the lines do intersect when $t=0$ and $s=-1 \Longrightarrow$ the point of interesction is $x=1, y=2, z=3$. or $P\,(1,2,3)$. A vector normal to the plane determine by these lines is
$\mathbf{n}_1\times\mathbf{n}_2=\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 4 \\ 1 & 2 & -4 \end{vmatrix}=-20\mathbf{i}+12\mathbf{j}+\mathbf{k}$, where n_1 and n_2 are directions of the lines $\Longrightarrow$ the plane containing the lines $\Longrightarrow$ the plane contain,ng the lines is represented by $(-20)\,(x-1)+(12)\,(y-2)+(1)\,(z-3)=0 \Longrightarrow -20x+12y+z=7$.

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28. Find the of intersection of the lines $x=t, y=-t+2, z=t+1$, and $x=2s+2, y=s+3, z=5s+6$,and then find the plane determined by these lines.
Answer:
the point of intersection is $x=0, y=2, z=1$. or $P\,(0,2,1)$.
the plane containing the lines $\Longrightarrow$ the plane containing the lines is represented by $(-6)\,(x-0)+(-3)\,(y-2)+(3)\,(z-1)=0 \Longrightarrow 6x+3y-3z=3$.

In Exercises 29 and 30, find the plane determined by the intersecting lines

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29.
$L1: x=-1+t, y=2+t, z=1-t,\ -\infty<t<\infty$

$L2: x = 1 - 4s, y = 1 + 2s, z = 2 - 2s, \ -\infty < s < \infty$
Solution:
The cross product of $\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $-4\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ has the same direction as the normal to the plane

$$\Longrightarrow \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 1 & -1 \\ -4 & 2 & -2 \end{vmatrix} = 6\mathbf{j} + 6\mathbf{k}\mathbf{n} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k},$$

Select a point on either line, such as $P(-1, 2, 1)$.

Since the lines are given to intersect, the desired plane is
$0(x + 1) + 6(y - 2) + (6)(z - 1) = 0 \Longrightarrow 6y + 6z = 18 \Longrightarrow y + z = 3.$

30.
$L1: x = t, y = 3 - 3t, z = 1 - t, \ -\infty < t < \infty$
$L2: x = 1 + s, y = 4 + s, z = -1 + s, \ -\infty < s < \infty$
Answer:

$$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & -1 \\ 1 & 1 & 1 \end{vmatrix} = -2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}.$$

Select a point on either line, such as $P(0, 3, -2)$.

Since the lines are given to intersect, the desired plane is
$(-2)(x - 0) + (-2)(y - 3) + (4)(z + 2) = 0 \Longrightarrow x + y - 2z = 7.$

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31. Find a plane through $P_0(2, 1, -1)$ and perpendicular to the line of intersection of the planes
$2x + y - z = 3, x + 2y + z = 2.$
Solution:

$$\mathbf{n}_1 \times \mathbf{n}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$$ is a vector in the direction of the line of intersection of the planes

$\Longrightarrow (3)(x - 2) + (-3)(y - 1) + (4)(z + 1) = 0 \Longrightarrow 3x - 3y + 3z = 0$ is the desired plane containing $P_0(2, 1, -1)$

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32. Find a plane through the points $P_1(1, 2, 3), P_2(3, 2, 1)$ and perpendicular to the plane $4x - y + 2z = 7$.
Solution:
A normal vector to the desired plane is

$$\overrightarrow{\mathbf{P}_1\mathbf{P}_2} \times \mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 2 \\ 4 & -1 & 2 \end{vmatrix} = -2\mathbf{i} - 12\mathbf{j} - 2\mathbf{k};$$

choosing $P_1(1, 2, 3)$ as point on the plane

$\Longrightarrow (-2)(x-1)+(-12)(y-2)+(-2)(z-3)=0 \Longrightarrow x+6y+z=16$ is the desired plane.

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3. Distances

In Exercises 33-38, find the distance from the point to the line.

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33. $(0,0,12)\,;\ x=4t, y=-2t, z=2t$
Solution:
$S(0,0,12)\,, P(0,0,0)$ and $\mathbf{v}=4\mathbf{i}-2\mathbf{j}+2\mathbf{k}$,

$$\overrightarrow{\mathbf{PS}}\times\mathbf{v}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & 12\\ 4 & -2 & 2\end{vmatrix}=24\mathbf{i}+48\mathbf{j}=24(\mathbf{i}+2\mathbf{j})$$

$$\Longrightarrow d=\frac{\left|\overrightarrow{\mathbf{PS}}\times\mathbf{v}\right|}{|\mathbf{v}|}=\frac{24\sqrt{1+4}}{\sqrt{16+4+4}}=\sqrt{(5)(24)}=2\sqrt{30}$$ is the distance from S to the line.

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34. $(0,0,0)\,;\ x=5+3t, y=5+4t, z=-3-5t$
Answer:
$d=3$

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35. $(2,1,3)\,;\ x=2+2t, y=1+6t, z=3$
Answer:
$d=0$
(i.e., the point lies on the line.)

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36. $(2,1,-1)\,;\ x=2t, y=1+2t, z=2t$
Answer:
$d=\sqrt{\dfrac{14}{3}}$

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37. $(3,-1,4)\,;\ x=4-t, y=3+2t, z=-5+3t$
Answer:
$d=\dfrac{9\sqrt{42}}{7}$

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38. $(-1,4,3)\,;\ x=10+4t, y=-3, z=4t$
Answer:
$d=7\sqrt{3}$

In Exercises 39-44, find the distance from the point to the plane.

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39. $(2,-3,4)\,;\ \ x+2y+2z=13$
Solution:

$S\,(2,-3,4)\,,\ x+2y+2z=13$ and $P\,(13,0,0)$ is on the plane
$\Longrightarrow \overrightarrow{\mathbf{PS}}=-11\mathbf{i}-3\mathbf{j}+4\mathbf{k}$ and $\mathbf{n}=\mathbf{i}+2\mathbf{j}+2\mathbf{k}$
$\Longrightarrow d=\left|\overrightarrow{\mathbf{PS}}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|=\left|\frac{-11-6+8}{\sqrt{1+4+4}}\right|=\left|\frac{-9}{\sqrt{9}}\right|=3$ iis the distance from S to the plane..

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40. $(0,0,0)\,;\ 3x+2y+6z=6$
Answer:
$d=\frac{6}{7}$

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41. $(0,1,1)\,;\ 4y+3z=-12$
Answer:
$d=\frac{19}{5}$

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42. $(2,2,3)\,;\ 2x+y+2z=4$
Answer:
$d=\frac{8}{3}$

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43. $(0,-1,0)\,;\ 2x+y+2z=4$
Answer:
$d=\frac{5}{3}$

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44. $(1,0,-1)\,;\ -4x+y+z=4$
Answer:
$d=\frac{3\sqrt{2}}{2}$

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45. Find the distance from the plane $x+2y+6z=1$ to the plane $x+2y+6z=10$.
Solution:
The point $P\,(1,0,0)$ is on the first plane and $S\,(10,0,0)$ is a point on the second plane $\Longrightarrow$ $\overrightarrow{\mathbf{PS}}=9\mathbf{i}$, and $\mathbf{n}=\mathbf{i}+2\mathbf{j}+6\mathbf{k}$ is normal to the first plane $\Longrightarrow$ the distance from S to the first plane is $d=\left|\overrightarrow{\mathbf{PS}}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|=\left|\frac{9}{\sqrt{1+4+36}}\right|=\frac{9}{41}$, which is also the distance between the planes.

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46. Find the distance from the line $x=2+t, y=1+t, z=-\frac{1}{2}-\frac{1}{2}t$ to the plane $x+2y+6z=10$.
Solution:
The line is parallel to the plane since $\mathbf{v}\cdot\mathbf{n}=\left(\mathbf{i}+\mathbf{j}-\frac{1}{2}\mathbf{k}\right)\cdot(\mathbf{i}+2\mathbf{j}+6\mathbf{k})=1+2-3=0$. Also the point $S\,(1,0,0)$ when $t=-1$ lies on the line, and the point $P\,(10,0,0)$ lies on the plane $\Longrightarrow \overrightarrow{\mathbf{PS}}=-9\mathbf{i}$. The distance from S to the plane is $d=\left|\overrightarrow{\mathbf{PS}}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|=\left|\frac{-9}{\sqrt{1+4+36}}\right|=\frac{9}{41}$, which is also the distance from the line to the plane.

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4. Angles

Find the angles between the planes in Exercises 47 and 48.

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47. $x + y = 1$, $2x + y - 2z = 2$.
Solution:
$\mathbf{n}_1 = \mathbf{i} + \mathbf{j}$ and $\mathbf{n}_2 = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k} \Longrightarrow \theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1|\,|\mathbf{n}_2|}\right) = \cos^{-1}\left(\frac{2+1}{\sqrt{2}\sqrt{9}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) =$
$\frac{\pi}{4}$

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48. $5x + y - z = 10$, $x - 2y + 3z = -1$.
Answer:
$\theta = \frac{\pi}{2}$

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5. Intersecting Lines and Planes

In Exercises 53-56, find the point in which the line meets the plane.

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53. $x = 1 - t,\ y = 3t,\ z = 1 + t$; $2x - y + 3z = 6$.
Solution:
$2x - y + 3z = 6 \Longrightarrow 2(1-t) - (3t) + 3z = 6 \Longrightarrow -2t + 5 = 6 \Longrightarrow t = -\frac{1}{2} \Longrightarrow x = \frac{3}{2}, y =$
$-\frac{3}{2}, z = \frac{1}{2}$
$\Longrightarrow$ the point is $\left(\frac{3}{2}, -\frac{3}{2}, \frac{1}{2}\right)$.

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54. $x = 2,\ y = 3 + 2t,\ z = -2 - 2t$; $6x + 3y - 4z = -12$.
Answer:
$\left(2, -\frac{20}{7}, \frac{27}{7}\right)$ is the point.

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55. $x = 1 + 2t,\ y = 1 + 5t,\ z = 3t$; $x + y + z = 2$.
Answer:
$(1, 1, 0)$ is the point

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56. $x = -1 + 3t,\ y = -2,\ z = 5t$; $2x - 3z = 7$.
Answer:
$(-4, -2, -5)$ is the point.

Find parametrizations for the lines in which the planes in Exercises 57-60 intersect.
57. $x +\ y + z = 1$; $x + y = 2$.
Solution:

$n_1 = i + j + k$ and $n_2 = i + j \Longrightarrow n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -i + j$, the direction of the desired line

$(1, 1, -1)$ is on both planes $\Longrightarrow$ the desired line is $x = 1 - t, y = 1 + t, z = -1$.

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58. $3x - 6y - 2z = 3$; $2x + y - 2z = 2$.
Answer:
$x = 1 + 14t, y = 2t, z = 1 + 3t$

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59. $x - 2y + 4z = 2$; $x + y - 2z = 5$.
Answer:
$x = 4,\ y = 3 + 6t,\ z = 1 + 3t$.

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60. $5x - 2y = 11,$; $4y - 5z = -17$.
Answer:
$x = 1 + 10t,\ y = -3 + 25t,\ z = 1 + 20t$.

Given two lines in space, either they are parallel, or they intersect, or they are skew (imagine for example, the flight paths of two planes in the sky). Exercises 61 and 62 each give three lines. In each exercise, determine whether the lines, taken two at a time, are parallel, intersect, or are skew. If they intersect, find the point of intersection.

61.
$L1 : x = 3 + 2t, y = -1 + 4t, z = 2 - t,\ -\infty < t < \infty$
$L2 : x = 1 + 4s, y = 1 + 2s, z = -3 + 4s,\ -\infty < s < \infty$
$L3 : x = 3 + 2r, y = 2 + r, z = -2 + 2r,\ -\infty < r < \infty$
Solution:
<u>L1&L2</u> $x = 3+2t = 1+4s$ and $y = -1+4t = 1+2s \Longrightarrow \begin{cases} 2t - 4s = -2 \\ 4t - 2s = 2 \end{cases} \Longrightarrow \begin{cases} 2t - 4s = -2 \\ 2t - s = 1 \end{cases}$
$-3s = -3 \Longrightarrow s = 1$ and $t = 1 \Longrightarrow$ on L1, $z = 1$ and on L2, $z = 1 \Longrightarrow$ L1 and L2

intersect at $(5, 3, 1)$.

<u>L2&L3</u>: The direction of L2 is $\frac{1}{6}(4\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}) = \frac{1}{3}(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ which is the same as the direction

$\frac{1}{3}(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ of L3; hence L2 and L3 are parallel.

<u>L1&L3</u> $x = 3 + 2t = 3 + 2r$ and
$y = -1+4t = 2+r \Longrightarrow \begin{cases} 2t - 2r = 0 \\ 4t - r = 3 \end{cases} \Longrightarrow \begin{cases} t - r = 0 \\ 4t - r = 3 \end{cases} \Longrightarrow 3t = 3 \Longrightarrow t = 1$ and $r = 1 \Longrightarrow$ on L1,
$z = 2$ while on L3, $z = 0 \Longrightarrow$ L1 and L3 do not intersect. the direction of L1 is $\frac{1}{\sqrt{21}}(2\mathbf{i} + 4\mathbf{j} - \mathbf{k})$

while the direction of L3 is $\frac{1}{3}(2i+j+2k)$ and neither is a multiple of the othe other; hence L1 and L3 are skew.

62.
$L1 : x = 1 + 2t, y = -1 - t, z = 3t, \ -\infty < t < \infty$
$L2 : x = 2 - s, y = 3s, z = 1 + s, \ -\infty < s < \infty$
$L3 : x = 5 + 2r, y = 1 - r, z = 8 + 3r, \ -\infty < r < \infty$
Answer:
L1 and L2 are skew; L2 and L3 intersect at $(1, 3, 2)$; L1 and L3 are parallel.

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65. Find the points in which the line $x = 1 + 2t, y = -1 - t, z = 3t$ meets the coordinate planes. Describe the reasoning behind your answer.
Solution:
$x = 0 \Longrightarrow t = -\frac{1}{2}, y = -\frac{1}{2}, z = \frac{-3}{2} \Longrightarrow \left(0, -\frac{1}{2}, -\frac{3}{2}\right);$
$y = 0 \Longrightarrow t = -1, x = -1, z = -3 \Longrightarrow (-1, 0, 3);$
$z = 0 \Longrightarrow t = 0, x = 1, y = -1 \Longrightarrow (1, -1, 0)$

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67. Is the line $x = 1 - 2t, y = 2 + 5t, z = -3t$ parallel to the plane $2x + y - z = 8$? Give reasons for your answer.
Solution:
With sustitution of the line into the plane we have $2(1 - 2t) + (2 + 5t) - (-3t) = 8 \Longrightarrow t = 1 \Longrightarrow$ the point $(-1, 7, -3)$ is contained in both the line and the plane, so they are not parallel.

68.

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69. Find two different planes whose intersection is the line $x = 1 + t, y = 2 - t, z = 3 + 2t$. Write equations for each plane in the form $Ax + By + Cz = D$.
Solution:
There are many possible answers. One found as follows: eliminate to get$t = x - 1 = 2 - y = \frac{z-3}{2} \Longrightarrow x - 1 = 2 - y$ and $2 - y = \frac{z-3}{2} \Longrightarrow x + y = 3$ and $2y + z = 7$ are two such planes.

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70. Find a plane through the origin that meets the plane $M : 2x + 3y + z = 12$ in a right angle. How do you know that your plane is perpendicular to M?
Solution:
Since the plane passes through the origin, its general equation is of the form $Ax + By + Cz = 0$. Since it meets the plane M at a right angle, their normal vectors are perpendicular $\Longrightarrow 2A + 3B + C = 0$. One choice satisfying this eqution is $A = 1, B = -1$ and $C = 1 \Longrightarrow x - y + z = 0$. Any plane $Ax + By + Cz = 0$ with $2A + 3B + C = 0$ will pass through the origin and be perpendicular to M

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71. For any nonzero numbers a, b, and c, the graph of $(x/a) + (y/b) = 1$ is a plane. Which planes have an equation of this form?

Solution:

The points $(a, 0, 0), (0, b, 0), (0, 0, c)$ are the x, y and z-intercepts of the plane. Since a, b, c are all nonzero, the plane must intersect all the coordinate axes and cannot pass through the origin. Thus $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ describes all planes <u>except</u> those through the origin or parallel to a coordinate axis.

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62. (p.901) A parallelogram has vertices $A(2, -1, 4), B(1, 0, -1), C(1, 2, 3)$, and D. Find

a. the coordinates of D

b. the cosine of the interior angle at B,

c. the vector projection of $\overrightarrow{BA}$ onto $\overrightarrow{BC}$,

d. the area of the parallelogram,

e. an equation for the plane of the parallelogram,

f. the areas of the orthogonal projections of the parallelogram on the three coordinate planes.

Solution:

(a) The line through A and B is $x = 1 + t, y = -t, z = -1 + 5t$; the line through C and D must be parallel and is

L_1:$x = 1 + t, y = 2 - t, z = 3 + 5t$. The line through B and C is $x = 1, y = 2 + 2s, z = 3 + 4s$; the line through A and D must be parallel and is

L_2:$x = 2, y = -1 + 2s, z = 4 + 4s$.

The lines L_1 and L_2 intersect at $D(2, 1, 8)$ where $t = 1$ and $s = 1$.

(b) $\cos\theta = \dfrac{(2\mathbf{j} + 4\mathbf{k})(\mathbf{i} - \mathbf{j} + 5\mathbf{k})}{\sqrt{20}\sqrt{27}} = \dfrac{3}{\sqrt{15}}$

(c) $\left(\dfrac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{\overrightarrow{BC} \cdot \overrightarrow{BC}}\right)\overrightarrow{BC} = \dfrac{18}{20}\overrightarrow{BC} = \dfrac{9}{5}(\mathbf{j} + 2\mathbf{k})$ where $\overrightarrow{BA} = \mathbf{i} - \mathbf{j} + 5\mathbf{k}$ and $\overrightarrow{BC} = 2\mathbf{j} + 4\mathbf{k}$

(d) area$= |(2\mathbf{j} + 4\mathbf{k}) \times (\mathbf{i} - \mathbf{j} + 5\mathbf{k})| = |14\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}| = 6\sqrt{6}$

(e) From part (d), $\mathbf{n} = 14\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ is normal to the plane $\Longrightarrow 14(x-1)+4(y-0)-2(z+1) = 0 \Longrightarrow 7x + 2y - z = 8$.

(f) From part (d), $\mathbf{n} = 14\mathbf{i}+4\mathbf{j}-2\mathbf{k} \Longrightarrow$ the area of the projection on the yz-plane is $|\mathbf{n} \cdot \mathbf{i}| = 14$; the area of the projection on the xy-plane is $|\mathbf{n} \cdot \mathbf{j}| = 4$; and the area of the projection on the xy-plane is $|\mathbf{n} \cdot \mathbf{k}| = 2$.
