ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science
MATH 237 Linear Algebra I
Final Exam Practice Problems D
January 7, 2008
13:00-14:50

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1. Let $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ be the linear transformation for which
$L(1,0,0) = (-2,-3)\,, L(0,1,0) = (3,2)\,, L(0,0,1) = (1,-1)$.
(a) Find $L(1,-1,2)$.

(b) Find $L(x_1, x_2, x_3)$.

(c) Determine the matrix of L relative to the standard bases $\mathbb{R}^3$ and $\mathbb{R}^2$.

(d) Determine the matrix of L relative to the standard basis for $\mathbb{R}^3$ and the basis $\{(2,3)\,,(3,2)\}$.

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2. Let L be the linear operator on the space of polynomials of degree ≤ 3 so that
$L(1) = 1+t, L(t) = t+t^2, L\left(t^2\right) = t^2+t^3, L\left(t^3\right) = 1$.
(a) Find $L\left(2-t+t^2-t^3\right)$.
(b) Compute the matrix of L relative to the basis $\mathcal{B} = \left\{1, t, t^2, t^3\right\}$.
(c) Find the matrix of L relative to the basis $\mathcal{C} = \left\{1+t, t+t^2, t^2+t^3, 1\right\}$
(d) Find the matrix of L relative to the pair $\mathcal{B}$,$\mathcal{C}$.

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3. Let
$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$.
Prove that the mapping given by
$L(X) = AX - XA$
is a linear operator on $\mathrm{M}_{2\times 2}(\mathbb{R})$. Find its matrix relative to the standard ordered basis for $\mathrm{M}_{2\times 2}(\mathbb{R})$.

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4. Let V be a finite dimensional vector space with bases $\mathcal{B}$,$\mathcal{B}'$ and let P, be the $\mathcal{B}$ to $\mathcal{B}'$ change of basis matrix. The operator $I : V \longrightarrow V$ given by $I(v) = v$ for all v in V is called the identity operator. Find the matrix of I
(a) relative to $\mathcal{C}$
(b) relative to $\mathcal{B}'$
(c) relative to $\mathcal{B}$ and $\mathcal{B}'$
(d) relative to $\mathcal{B}'$ and $\mathcal{B}$.

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5. Find the matrix of the projection (of $\mathbb{R}^3$) on the x_1x_2-plane relative to the standard basis.

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6. Find the matrix of the reflection (of $\mathbb{R}^3$) with respect to the x_1x_2-plane relative to the standard ordered basis.

7. Find the matrix of the reflection (of $\mathbb{R}^2$) with respect to the x_2-axis relative to the standard ordered basis.

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8. Find the matrix of the rotation (of $\mathbb{R}^2$) through the angle θ relative to the standard ordered basis.

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9. Let V be the vector space spanned by the UC-functions $\mathcal{B} = \{1, x, x^2, \cos x, \sin x\}$

a) Is $D = \dfrac{d}{dx}$ a linear operator on V? If so find its matrix relative to $\mathcal{B}$.

b) Is $\mathcal{J} = \displaystyle\int_0^x$ a linear operator on V? If so find its matrix relative to $\mathcal{B}$.

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10. Let $L : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be the linear operator given by

$$L(x_1, x_2, x_3) = x_1 - x_2 - x_3, -x_1 + x_2 - x_3, -x_1 - x_2 + x_3.$$

Find scalars λ and vectors v so that

$$L(v) = \lambda v.$$

Find a basis for $\mathbb{R}^3$ consisting of v's satisfying this condition and determine the matrix of L relative to this basis.
