ÇANKAYA UNIVERSITY

Department of Mathematics and Computer Science

MATH 237 Linear Algebra I

2$^{\text{nd}}$ Midterm
December 18, 2007
17:40-19:10

In Exercises 1 through 6 determine whether the given set, together with the given operations, is a vector space over $\mathbb{R}$ or not. If it is not a vector space, indicate the first axiom that fails to hold.

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1. The set of all smbols of the form $<< x, y >>$ where x and y are real numbers, together with the operations defined by

$$\begin{array}{rl} < & < x_1, y_1 >> + << x_2, y_2 >> = << x_1x_2, y_1y_2 >> \\ c \;\; < & < x, y >> = << c, xy >> \end{array}$$

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2. The set of symbols of the form $)x, y($ where x and y are real numbers together with the operations defined by

$$\begin{aligned})x_1, y_1(+)x_2, y_2(&=)x_1 + x_2, y_1 + y_2(\\ c)x_1, x_2(&=)cx_1, 0(\end{aligned}$$

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3. The set of positive real numbers together with operations $\oplus$ and $\odot$ defined by

$$\begin{aligned} p \oplus q &= pq \\ c \odot p &= p^c \end{aligned}$$

ere the tangent line is parallel to the line that passes through the points $(1, 1)$ and $(0, 3)$.

(b) Write an equation of the tangent line at each point found in part (a).

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4. $\mathbb{R}$ with ordinary addition and scalar multiplication.

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5. $\mathbb{C}$, the set of complex numbers with usual addition and multiplication by real numbers.

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6. 2×2 real matrices with operations

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} = \begin{bmatrix} a+a' & b+b' \\ c+c' & d+d' \end{bmatrix}$$
$$c \begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} cx & y \\ z & ct \end{bmatrix}$$

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7. Prove that in a vector space, V, there is only one element, z, satisfying $v+z=z+v=v$ for all v in V and for each v in V there is only one v' such that $v+v'=v'+v=0$.

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8. Is the subset

$$W = \{(a,b,c) \mid a,b,c \in \mathbb{Z}\}$$

of $\mathbb{R}^3$ closed under addition? Is it closed under scalar multiplication?

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9. Show that the subset $\{(x,y) : x,y \in \mathbb{R}, xy=0\}$ is not closed here the tangent line is parallel to the line that passes through the points $(1,1)$ and $(0,3)$.

(b) Write an equation of the tangent line at each point found in part (a).

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10. Prove that the following subsets of $\mathbb{R}^4$ are subspaces.

(a) $W_1 = \{(x,0,y,0) : x,y \in \mathbb{R}\}$.
(b) $W_1 = \left\{(x,y,z,t) \in \mathbb{R}^4 : x+y=z+t\right\}$.
(c) $W_1 = \{(x,x,y,0) : x,y \in \mathbb{R}\}$.

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11. Which of the following form subspaces of the vector space of the real $n \times n$ matrices?

(a) Upper triangular matrices.
(b) Diagonal matrices.
(c) Invertible matrices.
(d) Symmetric matrices.
(e) Skew-symmetric matrices.
(f) Singular matrices (that is, $\det = 0$).

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12. Show that the following form subspaces of the real vector space of real valued functions defined in $(-\infty,\infty)$

(a) The subset of all polynomial functions of the form $f(x) = a+bx$.
(b) Functions of the form $a\cos x + b\sin x$.
(c) Functions satisfying $y' + 2y = 0$.
(d) Even functions.
(e) Odd functions.
(f) Periodic functions of period π.

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13. Find the relation satisfied by a,b,c if (a,b,c) is a linear combination of

(a) $(1,-1,1),(2,1,-1),(4,-1,1)$.
(b) $(3,1,-2),(-2,1,1),(5,0,-3)$.

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14. Find k if $(k, 2k+1, 3k-1, k^2+3k-9)$ is in the subspace of $\mathbb{R}^4$ spanned by

$$(1,1,1,1),(-1,-1,-2,-1),(1,2,3,2) \text{ and } (1,0,4,2).$$

15. Determine the polynomial $1 + x + cx^2 + dx^3$ if it is a linear combination of

$$-1 + 2x + 3x^2 + 4x^3, 4 + x + 6x^2 + 11x^3, 1 - x - x^2 - x^3, \text{ and } 3 + x + 5x^2 + 9x^3.$$

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16. What must be a if $(-b, 4, 0, 2b - 2)$ is not contained in the subspace

$$< (-1, 1, -1, 1), (1, a, 1, -1), (-2a, -a, -2a + 1, 1 + 2a), (-1, 3, a - 1, 2) >?$$

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17. Can $(4, -2, 4)$ be written as a linear combination of $(1, 0, 1), (3, -4, 1)$, and $(4, -6, -2)$?

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18. Find the relations satisfied by a, b, and c if (a, b, c) is in the subspace of $\mathbb{R}^3$ spanned by $(1, -1, 0), (0, 1, 1)$, and $(3, -2, 1)$.

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19. Express $\{(1, -1, 0), (0, 1, 1), (3, -2, 1)\}$ as the solution space of a homogeneous system of linear equations.

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20. Express the space spanned by

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ 3 \\ 3 \end{bmatrix} \right\}$$

as the set of solutions of a homogeneous system.

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21. Find the value of x if

$$\begin{bmatrix} 4 & -4x - 2 \\ x + 10 & 4x + 6 \end{bmatrix} \in < \begin{bmatrix} 1 & -x \\ 3 & 1 + x \end{bmatrix}, \begin{bmatrix} -x & x^2 \\ -3x & -x^2 - x \end{bmatrix}, \begin{bmatrix} 2 & 2 - 3x \\ 9 & 3x + 1 \end{bmatrix} > .$$
