ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science

MATH 237
Linear Algebra I

1st Midterm
November 13, 2007
17:40-19:00

Surname : _____
Name : _____
ID # : _____
Department : _____
Section : _____
Instructor : _____
Signature : _____

- The exam consists of 6 questions.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might not get full credit.
- Calculators are not allowed.

GOOD LUCK!

Please do not write below this line.

Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
20	20	20	20	20	10	110

1. Given the system $\mathcal{S}$ of linear equations

$$\begin{aligned} x_1 + 2x_2 + ax_3 + 2x_4 &= 1 \\ x_1 + 3x_3 + 4x_4 &= b \\ 2x_1 + x_2 + (a+b)\,x_3 + 7x_4 &= 2 \end{aligned}$$

i) Find the value(s) of a and b so that the system has
   a) no solution;
   b) a unique solution;
   c) infinitely many solutions.

ii) Find all solutions in the case $a = 7, b = 1$.

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2.

a) Compute the following product

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ -5 & -4 & -3 & -2 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \end{bmatrix}$$

No explanation is necessary.

b) Let U be the matrix below. Find all solutions to the homogeneous system $Ux = 0$.

$$U = \begin{bmatrix} 1 & 1 & 1 & -2 & 0 \\ 0 & 0 & 1 & 7 & 5 \\ 0 & 0 & 0 & 0 & 7 \end{bmatrix}.$$

3. Compute the determinant

$$\det \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$

Note. You must show your work to receive credit for this problem.

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4. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 6 \\ 2 & 6 & c \end{bmatrix}$.

a) Find the number c that makes this matrix not invertible.
b) If $c = 20$ factor the matrix into $A = LU$ (lower triangular L and upper triangular U).
c) If $B^2 = 0$, the zero matrix, explain why B is not invertible.

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5. Let

$$A = \begin{bmatrix} 2 & 1 & -4 & 11 \\ 1 & -2 & -7 & 3 \\ -3 & 1 & 11 & -14 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -2 & -13 & 13 \\ 1 & 4 & 5 & 9 \\ -7 & 9 & 39 & -26 \end{bmatrix}.$$

a) Show that A and B have the same row reduced echelon form R.
b) Find invertible matrices P_1 and P_2 such that $R = P_1A$ and $R = P_2B$.
c) Find an invertible matrix P such that $A = PB$.

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6. (Bonus) Suppose the matrix A has row reduced echelon form R:

$$A = \begin{bmatrix} 1 & 2 & 1 & b \\ 2 & a & 1 & 8 \\ & \text{row } 3 & & \end{bmatrix}, \; R = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

a) What can you say about row 3 of A?
b) What are the numbers a and b?.