## **ÇANKAYA UNIVERSITY** Department of Mathematics and Computer Science

## MATH 237 Linear Algebra I

 $2^{nd}$  Midterm December 18, 2007 17:40-19:15

Surname	:	
Name	:	
ID #	:	
Department	•	
Section	•	
Instructor	•	
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Signature	:	

- The exam consists of 6 questions.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might <u>not</u> get full credit.
- $\bullet$  Calculators are <u>not</u> allowed.

## GOOD LUCK!

Please do <u>not</u> write below this line.

Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
20	20	20	20	20	10	110

**1.** Are the following sets in  $\mathbb{R}^3$  vector subspaces? Give reasons.

(a) 
$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : 2x - 2y + z = 0 \right\}$$
 **YES** **NO**

(b) 
$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : x^2 - y^2 + z = 0 \right\}$$
 **YES** NO

(c) 
$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : 2x - 2y + z = 1 \right\}$$
 **YES** NO

(d) 
$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : x = y \text{ AND } x = 2z \right\} \mathbf{\underline{YES}} \mathbf{NO}$$

(e) 
$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 : x = y \text{ OR } x = 2z \right\}$$
 **YES** NO

**2.** Find k if  $\alpha = (k, 2k+1, 3k-1, k^2+3k-9) \in \mathbb{R}^4$  is in the following subspace of  $\mathbb{R}^4$  span  $\{(1,1,1,1), (-1,-1,-2,-1), (1,2,3,2), (1,0,4,2)\}$ .

 $\{x + x^2, x - x^2 + x^3, 2 - x - x^3, x + 1\}.$ 

4. Consider the matrix

A =	1 1 1	2 2 2	$-1 \\ 0 \\ -1$	${0 \\ 2 \\ 0}$	$\begin{array}{c} 0\\ 2\\ 0 \end{array}$	
	$\begin{vmatrix} 1\\2 \end{vmatrix}$	$\frac{2}{4}$	$0^{-1}$	$\frac{1}{4}$	4	

(a) Find a basis for the row space of A.

(b) Find a basis for the nullspace N(A).

(b) Find a basis for the column space C(A).

## 5. Complete the set

 $\left\{ \left[ \begin{array}{cc} -1 & 1 \\ 1 & 1 \end{array} \right], \left[ \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right], \right\}$ 

to a basis of  $M_{2\times 2}(\mathbb{R})$ .

6. Find a basis and the dimension for the solution space of the system

$$2x - y + 3z + t = 0$$
  
-5x + y + 4z - t = 0  
-x - y + 10z + t = 0.