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Department of Mathematics and Computer Science

MATH 237
Linear Algebra I
First Midterm Practice Problems (A)

November 13, 2007
17:40 – 19:30

1. Find all solutions, if any exist, of the system

$$\begin{aligned} -3x+2y-z+t &= 5 \\ x-y+z-t &= 1 \\ 2x-3y+3t &= 0 \\ -x-y-z+4t &= 5 \\ x-4y-z+7t &= 5 \end{aligned}$$

2. Solve the following system if it is consistent:

$$\begin{aligned} x+y-z+t &= 2 \\ 2x-y+2z-t &= -1 \\ -x-4y+5z-4t &= 0 \\ x-y+4z-t &= 1 \end{aligned}$$

.

3. Find real numbers x, y, and z satisfying

$$x\begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix} + y\begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} + z\begin{bmatrix} 1 \\ 1 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 5 \end{bmatrix}.$$

4. Solve the system whose augmented matrix is

$$\left[\begin{array}{rrrrrr|r} 1 & 3 & -2 & 3 & -4 & 1 & 3 \\ 2 & 1 & -1 & -2 & 1 & 0 & 0 \\ -1 & 4 & 1 & -5 & 3 & 6 & 1 \\ 2 & 8 & -2 & -4 & 0 & 7 & 4 \end{array}\right].$$

5. Find all solutions of the following system over $\mathbb{Z}_3$:

$$\begin{aligned} x+y+z &= 1 \\ x+y+t &= 0 \\ y+z+t &= 1 \\ y+t &= 1 \\ x+z+t &= 0 \end{aligned}$$

.

6. Solve

$$\begin{aligned} x+y-z+t &= 2 \\ 2x-y+2z-t &= -1 \\ -x-4y+5z-4t &= 0 \\ x-y+4z-t &= 1 \end{aligned}$$

over $\mathbb{Z}_7$.

7. What must be the value of a if there are numbers x, y, and z such that

$$x\begin{bmatrix} 1 \\ -1 \\ 3 \\ 2 \end{bmatrix} + y\begin{bmatrix} 2 \\ 1 \\ -1 \\ 1 \end{bmatrix} + z\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a+1 \\ a \\ -1 \\ -a \end{bmatrix}.$$

8. Find the values of a and b for which the system

$$\begin{aligned} 2z-y+2az+t &= b \\ -2x+ay-3z &= 4 \\ 2z-y+(2a+1)\,z+(a+1)\,t &= 0 \\ -2x+y4\,(1-2a)\,z-2t &= -2b-2 \end{aligned}$$

has
(a) A unique solution.
(b) Infinitely many solutions.
(c) No solution.

9. Solve the homogeneous system

$$\begin{aligned} -x+2y+z-2t &= 0 \\ -x-y+z+t &= 0 \\ x+y+z+t &= 0 \\ -x+5y+4z-5t &= 0 \\ 2x-y+z+t &= 0 \end{aligned}$$

.

10. Find the general solution of the following homogeneous system in terms of its fundamental solutions:

$$\begin{aligned} x+y-z+t+u &= 0\\ x+2y-2z+3t &= 0\\ -x-2y+4z-7t+u &= 0\\ x-t+u &= 0\\ 2x+2y-2z+2t+u &= 0 \end{aligned}$$

.

11. Find fundamental solutions of

$$\begin{aligned} x_1+x_2-x_3-x_4+x_5 &= 0\\ 2x_1+2x_2-x_4+x_5 &= 0\\ x_1+x_2+x_3-2x_4+x_5 &= 0\\ -x_1-x_2+3x_3+2x_4-2x_5 &= 0\\ 2x_3-x_4 &= 0\\ x_1+x_2+5x_3+2x_4-2x_5 &= 0 \end{aligned}$$

.

12. Find fundamental solutions of homogeneous systems whose coefficient matrices are

$$\text{a)} \begin{bmatrix} 0 & 0 & 0 & 3 & -4 & 1 & 3\\ 0 & 0 & 0 & -2 & 1 & 0 & 0\\ 0 & 0 & 0 & -5 & 5 & -1 & -3\\ 0 & 0 & 0 & -4 & 2 & 0 & 0 \end{bmatrix} \quad \text{b)} \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -2 & 3 & -1\\ 0 & 0 & 0 & 0 & 2 & -4 & 6 & -3\\ 0 & 0 & 0 & 0 & 1 & -2 & 1 & 2\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix}.$$

13. What must be the values of t if the following homogeneous system has nontrivial solutions?

$$\begin{aligned} x_1+tx_2-3x_4 &= 0\\ (t-1)\,x_1-(t+1)\,x_2+3x_4 &= 0\\ x_1+tx_2+(t+2)\,x_3-3x_4 &= 0\\ (t+1)\,x_1+tx_2+(t+5)\,x_3-3x_4 &= 0 \end{aligned}$$

14. What must be k if the matrix

$$\begin{bmatrix} -1 & k & 3 & -2\\ 2 & 1 & 1 & k\\ 1 & k+1 & 4 & k-2\\ 2 & 1 & 1 & k+1 \end{bmatrix}$$

is invertible?

15. Prove:

(a) An $m \times n$ matrix with $m < n$ has no left inverse.

(b) An $m \times n$ matrix with $m > n$ has no right inverse.

Deduce that a matrix which has a right inverse and a left inverse should be a (square) invertible matrix.

16. Prove that if A is an $n \times n$ matrix which is not invertible then there is a square matrix, B, with at leat n nonzero entries such that $AB = 0$. Determine all such matrices over $\mathbb{Z}_3$ for

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

.

17. Show that

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

has no LU- decomposition.