



Your Name / İsim Soyisim

Your Signature / İmza

Student ID # / Öğrenci Numarası

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Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- Give your answers in exact form (for example  $\frac{\pi}{3}$  or  $5\sqrt{3}$ ), except as noted in particular problems.
- Calculators, cell phones are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Place a box around your answer to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 70 min.

| Problem | Points | Score |
|---------|--------|-------|
| 1       | 20     |       |
| 2       | 20     |       |
| 3       | 20     |       |
| 4       | 20     |       |
| 5       | 20     |       |
| Total:  | 100    |       |

Do not write in the table to the right.

1. 20 points Suppose that  $\begin{bmatrix} 1 & 0 & k^2 \\ 0 & 1 & 4 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ k^2 \\ 2 \end{bmatrix}$ .

- For what values of  $k$  does the linear system have a unique solution ?
- For what values of  $k$  does the linear system have infinitely many solutions ?
- For what values of  $k$  does the linear system have no solution ?

2. 20 points Let  $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 2 & 3 & -1 & 4 & 2 \\ 4 & -1 & 0 & 0 & -1 \\ 2 & 3 & -1 & 2 & 2 \\ -1 & -2 & 0 & 3 & 4 \end{bmatrix}$  Calculate the determinant of  $\mathbf{A}$ ,  $\det \mathbf{A}$ , by using row or column expansion.

3. 20 points Find the set of all solutions of the following system of linear equations.

$$3w + 3x + 5z = 0$$

$$-x + y - 3z = 0$$

$$2w - x + 3y - z = 0$$

$$-3w + x - 4y + 5z = 0$$

4. 20 points Calculate the

$$\det \mathbf{A} = \begin{vmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{vmatrix}$$

5. 20 points Suppose that  $(3A^T)^{-1} = \begin{bmatrix} 1 & 2 & -3 \\ 4 & -1 & 2 \\ 2 & 2 & -3 \end{bmatrix}$ . Find the matrix  $A$ .