Your Name

Your Signature

Student ID #

Professor's Name

Your Department

- This exam is closed book.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- Calculators, cell phones are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 3 pages, plus this cover sheet. Please make sure that your exam is complete.

Problem	Points	Score
1	15	
2	20	
3	10	
4	15	
5	15	
6	15	
7	10	
Total:	100	

Do not write in the table to the right.

Question:	1	2	3	4	5	6	7	Total
Points:	15	20	10	15	15	15	10	100
Score:								

1. (15 total points) Given the function $f(x) = \dfrac{x^2+6x+5}{x+5}$, and point $x_0 = -5$ and $\varepsilon = 0.05$ p.64, pr.34

   (a) (5 Points) Find $L = \lim_{x\to -5} f(x)$. (DO NOT USE L'HÔPITAL's RULE)

$$\begin{aligned}\lim_{x\to -5}\frac{x^2+6x+5}{x+5} &= \lim_{x\to -5}\frac{\cancel{(x+5)}(+x+1)}{\cancel{(x+5)}}\\ &= \lim_{x\to -5}(x+1)\\ &= -4, \quad x\neq -5.\end{aligned}$$

   (b) (10 Points) Find a number δ such that for all x

$$0 < |x-x_0| < \delta \Rightarrow |f(x-L)| < \varepsilon.$$

Step 1

$$\begin{aligned}\left|\left(\frac{x^2+6x+5}{x+5}\right)-(-4)\right| &< 0.05 \Longrightarrow -0.05 < \frac{x^2+6x+5}{x+5}+4 < 0.05\\ &\Longrightarrow -4.05 < x+1 < -3.95,\ x\neq -5\\ &\Longrightarrow -5.05 < x < -4.95,\ x\neq -5\end{aligned}$$

Step 2

$$\begin{aligned}|x-(-5)| &< \delta \Longrightarrow -\delta < x+5 < \delta \Longrightarrow -\delta-5 < x < \delta-5.\\ \text{Then } -\delta-5 &= -5.05 \Longrightarrow \delta = 0.05,\ \text{or } \delta-5 = -4.95 \Longrightarrow \delta = 0.05;\ \text{thus } \delta = 0.05.\end{aligned}$$

2. Evaluate the following limits.

   (a) (10 Points) $\lim_{x\to 1}\dfrac{x^{1/3}-1}{\sqrt{x}-1}$ (DO NOT USE L'HÔPITAL's RULE)

$$\begin{aligned}\lim_{x\to 1}\frac{x^{1/3}-1}{\sqrt{x}-1} &= \lim_{x\to 1}\frac{x^{1/3}-1}{\sqrt{x}-1}\,\frac{(x^{2/3}+x^{1/3}+1)(\sqrt{x}+1)}{(x^{2/3}+x^{1/3}+1)(\sqrt{x}+1)}\\ &= \lim_{x\to 1}\frac{\cancel{(x-1)}(\sqrt{x}+1)}{\cancel{(x-1)}(x^{2/3}+x^{1/3}+1)}\\ &= \lim_{x\to 1}\frac{\sqrt{x}+1}{x^{2/3}+x^{1/3}+1}\\ &= \frac{1+1}{1+1+1}\\ &= \frac{2}{3}\end{aligned}$$

   (b) (10 Points) $\lim_{x\to 0}\dfrac{x\sin x}{2-2\cos x}$ p.178 pr.101 (DO NOT USE L'HÔPITAL's RULE)

$$\begin{aligned}\lim_{x\to 0}\frac{x\sin x}{2-2\cos x} &= \lim_{x\to 0}\frac{x\sin x}{2(1-\cos x)} = \lim_{x\to 0}\frac{x\sin x}{2(2\sin^2(\frac{x}{2}))} = \lim_{x\to 0}\left[\frac{\frac{x}{2}\frac{x}{2}}{\sin^2(\frac{x}{2})}\cdot\frac{\sin x}{x}\right]\\ &= \lim_{x\to 0}\left[\frac{\frac{x}{2}}{\sin(\frac{x}{2})}\cdot\frac{\frac{x}{2}}{\sin(\frac{x}{2})}\,\frac{\sin x}{x}\right]\\ &= (1)(1)(1) = 1.\end{aligned}$$

3. (10 Points) Find $\dfrac{dy}{dt}$ if $y = (t^{-3/4}\sin t)^{4/3}$.

p.147 pr.46

$$\begin{aligned} \frac{dy}{dt} &= \frac{d}{dt}\left(t^{-3/4}\sin t\right)^{4/3} \\ &= \frac{4}{3}\left(t^{-3/4}\sin t\right)^{1/3}[-\frac{3}{4}t^{-7/4}\sin t \\ &\quad +t^{-3/4}\cos t] \end{aligned}$$

4. (15 Points) Find the equations of normals to the curve

$$xy + 2x - y = 0$$

that are parallel to the line $2x + y = 0$. p.154, pr.40

$$\begin{aligned} xy + 2x - y &= 0 \Longrightarrow x\frac{dy}{dx} + y + 2 - \frac{dy}{dx} = 0 \\ &\Longrightarrow \frac{dy}{dx} = \frac{y+2}{1-x}; \end{aligned}$$

the slope of the line $2x + y = 0$ is -2. In order to be parallel, the normal lines must also have slope of -2. Since a normal is perpendicular to a tangent, the slope of tangent is $\frac{1}{2}$. Therefore

$$\begin{aligned} \frac{y+2}{1-x} &= \frac{1}{2} \Longrightarrow 2y + 4 = 1 - x \\ &\Longrightarrow x = -3 - 2y. \end{aligned}$$

Substituting in the original equation,

$$\begin{aligned} y(-3-2y) + 2(-3-2y) &= 0 \\ &\Longrightarrow y^2 + 4y + 3 = 0 \\ &\Longrightarrow y = -3 \text{ or } y = -1. \end{aligned}$$

$$\begin{aligned} \text{If } y &= -3, \text{then } x = 3 \text{ and } y + 3 = -2(x-3) \\ &\Longrightarrow y = -2x + 3. \\ \text{If } y &= -1, \text{ then } x = -1 \text{ and } y + 1 = -2(x+1) \\ &\Longrightarrow y = -2x - 3. \end{aligned}$$

5. (15 Points) For what values of a and b is

$$f(x) = \begin{cases} -2 & x \le -1 \\ ax - b, & -1 < x < 1 \\ 3, & x \ge 1 \end{cases}$$

continuous at every x. p.83, pr.45

Clearly f is continuous if $x \ne -1$ and for $x \ne 1$ for if $x < -1$ or if $-1 < x < 1$ or if $x > 1$, f is a polynomial, regardless the values of a and b. For continuity at $x = -1$, we require that the one-sided limits of $f(x)$ at $x = -1$ be equal. But $\lim\limits_{x\to -1^-} f(x) = -2$ and $\lim\limits_{x\to -1^+} f(x) = a(-1) + b = -a + b$.
Similarly, for continuity at $x = 1$, we require that the one-sided limits of $f(x)$ at $x = 1$ be equal. But $\lim\limits_{x\to 1^-} f(x) = a(1) + b = a + b$ and $\lim\limits_{x\to 1^+} f(x) = 3$.
Equality of one-sided limits is equivalent to

$$\begin{aligned} -2 &= -a + b \text{ and } a + b = 3 \\ &\Longrightarrow a = \frac{5}{2} \text{ and } b = \frac{1}{2}. \end{aligned}$$

6. (15 Points) Assume that $f(x)$ and $g(x)$ are differentiable functions satisfying

$$\begin{array}{llll} g(0)=1 & f(0)=1 & f(1)=3 & g(1)=5 \\ g'(0)=\frac{1}{2} & f'(0)=-3 & f'(1)=\frac{1}{2} & g'(1)=-4 \end{array}$$

Let $h(x)=f(x+g(x))$. Evaluate $h'(0)$. p.148, pr.74

First, by the Chain Rule, we have $h'(x)=f'(x+g(x))(1+g'(x))$. Then $h'(0)=f'(0+g(0))(1+g'(0))=f'(g(0))(1+g'(0))$.
Hence $h'(0)=f'(1)(1+\dfrac{1}{2})=(\dfrac{1}{2})(\dfrac{3}{2})=\dfrac{3}{4}$.

7. (10 Points) Find the value or values of c that satisfy the equation

$$\frac{f(b)-f(a)}{b-a}=f'(c)$$

in the conclusion of the Mean Value Theorem for the function $f(x)=x+\dfrac{1}{x}$ and interval $[\dfrac{1}{2},2]$. p.196, pr.3

When $f(x)=x+\dfrac{1}{x}$ for $\dfrac{1}{2}\le x\le 2$, then

$$\frac{f(2)-f(1/2)}{2-1/2}=f'(c)\Rightarrow 0=1-\frac{1}{c^2}\Rightarrow c=\pm 1$$

But $-1\notin[\frac{1}{2},2]$, so $c=1$.