Your Name / Adınız - Soyadınız

Signature / İmza

Student ID # / Öğrenci No

(**mavi tükenmez!**)

Soru	1	2	3	4	5	6	7	Toplam
Maks.	14	11	14	12	13	14	20	98
Puanınız								

1. (14 points) $\lim_{x\to 64} \frac{x^{2/3}-16}{\sqrt{x}-8} = ?$

Solution:

$$\begin{aligned}
\lim_{x\to 64} \frac{x^{2/3}-16}{\sqrt{x}-8} &= \lim_{x\to 64} \frac{(x^{1/3}-4)(x^{1/3}+4)}{\sqrt{x}-8} \\
&= \lim_{x\to 64} \frac{(x^{1/3}-4)(x^{1/3}+4)}{\sqrt{x}-8} \cdot \frac{(x^{2/3}+4x^{1/3}+16)(\sqrt{x}+8)}{(x^{2/3}+4x^{1/3}+16)(\sqrt{x}+8)} \\
&= \lim_{x\to 64} \frac{\cancel{(x-64)}(x^{1/3}+4)(\sqrt{x}+8)}{\cancel{(x-64)}(x^{2/3}+4x^{1/3}+16)} = \lim_{x\to 64} \frac{(x^{1/3}+4)(\sqrt{x}+8)}{(x^{2/3}+4x^{1/3}+16)} \\
&= \frac{(4+4)(8+8)}{16+16+16} = \boxed{\frac{8}{3}}
\end{aligned}$$

p.97, pr.18

2. (11 Puan) $\lim_{x\to 0} \frac{x+x\cos x}{\sin x \cos x} = ?$

Solution:

$$\begin{aligned}
\lim_{x\to 0} \frac{x+x\cos x}{\sin x\cos x} &= \lim_{x\to 0} \left(\frac{x}{\sin x \cos x} + \frac{x\cancel{\cos x}}{\sin x \cancel{\cos x}} \right) \\
&= \lim_{x\to 0} \left(\frac{x}{\sin x} \frac{1}{\cos x} \right) + \lim_{x\to 0} \left(\frac{x}{\sin x} \right) \\
&= \lim_{x\to 0} \left(\frac{1}{\frac{1}{\sin x}} \right) \cdot \lim_{x\to 0} \left(\frac{1}{\cos x} \right) + \lim_{x\to 0} \left(\frac{1}{\frac{1}{\sin x}} \right) = (1)(1)+1 = \boxed{2}
\end{aligned}$$

p.73, pr.29

3. Kapalı tanımlı $x\sin 2y = y\cos 2x$ eğrisi verilsin.

   (a) (7 Puan) Eğriye $(\pi/4, \pi/2)$ noktasındaki teğetin denklemini yazınız.

Solution:

$$x\sin 2y = y\cos 2x \Rightarrow x(\cos 2y)y' + \sin 2y = -2y\sin 2x + y'\cos 2x \Rightarrow y'(2x\cos 2y - \cos 2x) = -\sin 2y - y\sin 2x$$

Hence

$$y' = \frac{\sin 2y + 2y\sin 2x}{\cos 2x - 2x\cos 2y};$$

the slope of the tangent line $m = y'\big|_{(\pi/4,\pi/2)} = \left.\frac{\sin 2y + 2y\sin 2x}{\cos 2x - 2x\cos 2y}\right|_{(\pi/4,\pi/2)} = \frac{\pi}{\frac{\pi}{2}} = 2 \Rightarrow$

the tangent line is $y - \frac{\pi}{2} = 2(x - \frac{\pi}{4}) \Rightarrow \boxed{y = 2x.}$

p.154, pr.36

(b) (7 Puan) Eğriye $(\pi/4, \pi/2)$ noktasındaki normalin denklemini yazınız.

Solution: the normal line is $y - \frac{\pi}{2} = -\frac{1}{2}(x - \frac{\pi}{4}) \Rightarrow \boxed{y = -\frac{1}{2}x + \frac{5\pi}{8}.}$

p.154, pr.36

4. (12 Puan) $\lim_{x\to 23} \sqrt{x-7} = 4$ olduğunu $\varepsilon = 1$ için doğrulayan bir $\delta > 0$ bulunuz.

Solution:

$$\left|\sqrt{x-7} - 4\right| < 1 \Rightarrow -1 < \sqrt{x-7} - 4 < 1 \Rightarrow 3 < \sqrt{x-7} < 5 \Rightarrow 9 < x-7 < 25 \Rightarrow 16 < x < 32$$

Now we have

$$|x-23| < \delta \Rightarrow -\delta < x-23 < \delta \Rightarrow -\delta + 23 < x < \delta + 23.$$

Then

$$-\delta + 23 = 16 \Rightarrow \delta = 7, \text{ or } \delta + 23 = 32 \Rightarrow \delta = 9; \text{ or } \boxed{\delta = \min\{7, 9\} = 7.}$$

p.94, pr.33

5. (13 Puan) a ve b nin hangi değerleri için

$$f(x) = \begin{cases} -2, & x \le 1 \\ ax - b, & -1 < x < 1 \\ 3, & x \ge 1 \end{cases}$$

her x noktasında süreklidir?

Solution:

$$\begin{aligned}
\lim_{x\to -0} f(x) &= -2 \text{ and} \\
\lim_{x\to 1^+} f(x) &= a(-1) + b = -a + b, \\
\text{and } \lim_{x\to 1} f(x) &= a(1) + b = a + b \\
\text{and } \lim_{x\to 1^+} f(x) &= 3. \\
& \text{For } f(x) \text{ to be continuous} \\
\text{we must have } -2 &= -a + b \text{ and } a + b = 3 \\
\Longrightarrow & \boxed{a = \frac{5}{2} \text{ and } b = \frac{1}{2}.}
\end{aligned}$$

p.94, pr.33

6. (14 Puan) *Sadece türev tanımı kullanarak*, $s = t^3 - t^2$ ise $t = -1$ de $\frac{ds}{dt}$ yi bulunuz.

Solution:

$$\begin{aligned}
\frac{ds}{dt} = \lim_{h\to 0} \frac{\left[(1+h)^3 - (1+h)^2\right] - (t^3 - t^2)}{h} &= \lim_{h\to 0} \frac{(t^3 + 3t^2h + 3th^2 + h^3) - (t^2 + 2th + h^2) - t^3 + t^2}{h} \\
&= \lim_{h\to 0} \frac{3t^2h + 3th^2 + h^3 - 2th - h^2}{h} \\
&= \lim_{h\to 0} \frac{h(3t^2 + 3th + h^2 - 2t - h)}{h} \\
&= \lim_{h\to 0} (3t^2 + 3th + h^2 - 2t - h) \\
&= 3t^2 - 2t; \left.\frac{ds}{dt}\right|_{t=-1} = \boxed{5}
\end{aligned}$$

p.112, pr.15

7. (a) (10 Puan) $y = \left(\frac{3t-4}{5t+2}\right)^{-5}$ ise türevini bulunuz.

Solution:

$$y = \left(\frac{3t-4}{5t+2}\right)^{-5} \Rightarrow \frac{dy}{dt} = -5\left(\frac{3t-4}{5t+2}\right)^{-6} \cdot \frac{(5t+2)(3)-(3t-4)(5)}{(5t+2)^2} = -5\left(\frac{5t+2}{3t-4}\right)^{-6} \frac{\cancel{15t}+6-\cancel{15t}+20}{(5t+2)^2} = \boxed{\frac{-130(5t+2)^4}{(3t-4)^{(}6)}}$$

p.147, pr.48

(b) (10 Puan) $f(u) = u + \frac{1}{\cos^2 u}$ ve $u = g(x) = \pi x$ ise $(f \circ g)'$ nin $x = \frac{1}{4}$ deki değerini bulunuz.

Solution:

$$g(x) = \pi x \Rightarrow g'(x) = \pi \Rightarrow g\left(\frac{1}{4}\right) = \frac{\pi}{4} \text{ and } g'\left(\frac{1}{4}\right) = \pi.$$

Also

$$f(u) = u + \sec^2 u \Rightarrow f'(u) = 1 + 2\sec u \cdot \sec u \tan u = 1 + 2\sec^2 u \tan u \Rightarrow f'\left(g\left(\frac{1}{4}\right)\right) = f'\left(\frac{\pi}{4}\right) = 1 + 2\sec^2\frac{\pi}{4}\tan\frac{\pi}{4} = 5.$$

Therefore,

$$(f \circ g)'\left(\frac{1}{4}\right) = f'\left(g\left(\frac{1}{4}\right)\right) \cdot g'\left(\frac{1}{4}\right) = \boxed{5\pi}.$$

p.147, pr.68