Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- This exam is closed book.
- Give your answers in exact form (for example $\frac{\pi}{3}$ or $5\sqrt{3}$), except as noted in particular problems.
- Calculators, cell phones are not allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Do not ask the invigilator anything.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 70 min.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total:	100	

Do not write in the table to the right.

1. (a) $\boxed{\text{10 Points}}$ Find a function $y = f(x)$ satisfying $f'(x) = 9x^2 - 4x + 5$ and $f(-1) = 0$

Solution: $f'(x) = 9x^2 - 4x + 5$ implies $f(x) = 3x^3 - 2x^2 + 5x + C$. We must find the value of C. For that purpose plug $x = -1$ and $y = 0$. Hence

$$0 = 3(-1)^3 - 2(-1)^2 + 5(-1) + C \Rightarrow 0 = -3 - 2 - 5 + C \Rightarrow C = 10.$$

Therefore, the function with the required property is

$$f(x) = 3x^3 - 2x^2 + 5x + 10.$$

Now it is easy to see that this is the actual function we wanted to find.

p.695, pr.31

(b) $\boxed{\text{10 Points}}$ Find the *average value* of $f(t) = t^2 - t$ over the interval $[-2, 1]$.

Solution:

$$\begin{aligned}
av(f) &= \left(\frac{1}{1-(-2)}\right)\int_{-2}^{1}(t^2 - t)\,dt \\
&= \frac{1}{3}\int_{-2}^{1} t^2\,dt - \frac{1}{3}\int_{-2}^{1} t\,dt \\
&= \frac{1}{3}\int_{0}^{1} t^2\,dt - \frac{1}{3}\int_{0}^{-2} t^2\,dt - \frac{1}{3}\left(\frac{1^2}{2} - \frac{(-2)^2}{2}\right) \\
&= \frac{1}{3}\left(\frac{(1)^3}{3}\right) - \frac{1}{3}\left(\frac{(-2)^3}{3}\right) + \frac{1}{2} = \boxed{\frac{3}{2}}
\end{aligned}$$

p.452, pr.24

2. (a) $\boxed{\text{7 Points}}$ Use the approximation $(1+x)^k \approx 1+kx$ to estimate $(1.0002)^{50}$.

Solution: Letting $x = 00002$ and $k = 50$, we have

$$(1.0002)^{50} = (1+0.0002)^{50} \approx 1+50(0.0002) = 1+.01 = \boxed{1\cdot 01.}$$

p.55, pr.36

(b) $\boxed{\text{6 Points}}$ Find the *differential* dy if $2y^{3/2}+xy-x=0$.

Solution:

$$\begin{aligned}2y^{3/2}+xy-x=0 &\Rightarrow 3y^{1/2}dy+ydx+xdy-dx=0\\ &\Rightarrow (3y^{1/2}+x)dy=(1-y)dx\\ &\Rightarrow \boxed{dy=\frac{1-y}{3y^{1/2}+x}\,dx}\end{aligned}$$

p.73, pr.38

(c) $\boxed{\text{7 Points}}$ The radius of a circle is increased from 2.00 to 2.02 m. Estimate the resulting change in area.

Solution: Given $r = 2\text{m}$, $dr = .2$.

$$A = \pi r^2 \Rightarrow dA = 2\pi r\,dr = 2\pi(2)(.02) = 0.08m^2$$

p.73, pr.38

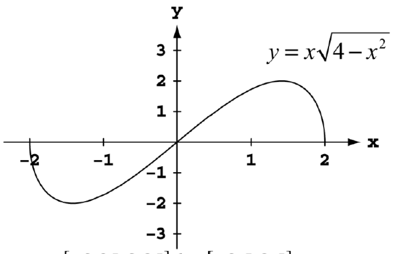
3. (a) $\boxed{\text{14 Points}}$ Find all the local and absolute extreme values of $y = x\sqrt{4-x^2}$.

Solution: First, the derivative is

$$y' = x\frac{1}{2\sqrt{4-x^2}}(-2x)+(1)\sqrt{4-x^2} = \frac{-x^2+(4-x^2)}{\sqrt{4-x^2}} = \frac{4-2x^2}{\sqrt{4-x^2}}.$$

Then the critical points occur when $4-2x^2=0$ and when $4-x^2=0$. That is, the critical points are ± 2 and $\pm\sqrt{2}$.

crit. pt.	derivative	extremum	value
$x = -2$	undefined	local max	0
$x = -\sqrt{2}$	0	minimum	-2
$x = \sqrt{2}$	0	maximum	2
$x = 2$	undefined	local min	0



$[-2.35, 2.35]$ by $[-3.5, 3.5]$

p.83, pr.38

(b) $\boxed{\text{6 Points}}$ Show that $r(\theta) = \theta + \sin^2\left(\frac{\theta}{3}\right) - 8$ has exactly one zero in $(-\infty,\infty)$. Give reason.

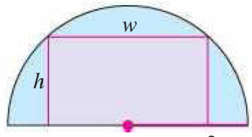
Solution: First, notice that

$$r'(\theta) = 1+\frac{2}{3}\sin\left(\frac{\theta}{3}\right)\cos\left(\frac{\theta}{3}\right) = 1+\frac{1}{3}\sin\left(\frac{2\theta}{3}\right) > 0$$

on $(-\infty,\infty)$. So $r(\theta)$ is increasing on $(-\infty,\infty)$. Further $r(0) = -8$ and $r(8) = \sin^2\left(\frac{8}{3}\right) > 0$ implies that $r(\theta)$ has exactly one zero in $(-\infty,\infty)$.

p.487, pr.12

4. (a) $\boxed{\text{10 Points}}$ Determine the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 3.



Solution: Let h be the length of the side of the rectangle which is perpendicular to the straight-edge of the semicircle, and w be the length of the side which lies along the straight-edge. Draw a couple of radii of the circle and observe that $w/2 = \sqrt{3^2 - h^2}$, so $w = 2\sqrt{9-h^2}$. Thus, the area of the rectangle we wish to maximize is given by

$$A = hw = h = 2h\sqrt{9-h^2} = 2\sqrt{9h^2-h^4}$$
$$\Rightarrow \frac{dA}{dh} = \frac{2(18h-4h^3)}{\sqrt{9h^2-h^4}} = \frac{4h(9-2h^2)}{h\sqrt{9-h^2}} = \frac{4(9-2h^2)}{\sqrt{9-h^2}}$$

Thus, A has a critical point when $9-2h^2 = 0$, or when $h^2 = 9/2$ which is equivalent to $h = 3/\sqrt{2}$. Again, verify that this is indeed a maximum (using either the first or second derivative test), and then we see that the dimensions of the rectangle maximizing area are $h = 3/\sqrt{2}$ and

$$w == 2\sqrt{9-(3/\sqrt{2})^2} = 3\sqrt{2}.$$

The function A is continuous on the closed interval $0 \le h \le 3$ and so has an absolute maximum and an absolute minimum on this interval.

$$A(0) = 0 \quad (\text{ ABS MIN.}) \qquad A(3) = 0 \quad (\text{ ABS MIN.}) \qquad A(3/\sqrt{2}) = (3\sqrt{2})(3/\sqrt{2}) = \boxed{9} \quad (\text{ ABS MAX.})$$

p.695, pr.29

(b) $\boxed{\text{10 Points}}$ What values of a and b make $f(x) = x^3 + ax^2 + bx$ have a local minimum at $x = -1$ and a point of inflection $x = 1$?.

Solution: First the derivatives are

$$f'(x) = 3x^2 + 2ax + b \qquad \text{and} \qquad f"(x) = 6x + 2a.$$

Suppose that f has a local minimum at $x = -1$. Then we must have $f'(-1) = 0$. Hence we must have

$$3(-1)^2 + 2a(-1) + b = 0 \Rightarrow 3 - 2a + b = 0 \Rightarrow 2a - b = 3.$$

Further, suppose that f has a point of inflection at $x = 1$. Then $f''(1) = 0$ must be true. Hence $6 + 2a = 0$ which implies $a = -3$. Now we have $2(-3) - b = 3 \quad b = -9$. Hence the function we may need to search for, is $f(x) = x^3 - 3x^2 - 9x$. It remains to check if this is the correct function we want. The derivatives are $f'(x) = 3x^2 - 6x - 9 = 3(x^2 - 2x - 3) = 3(x-3)(x+1)$ and $f''(x) = 6x - 6$. If $x < 1$, we have $f''(x) < 0$ and if $1 < x$, we have $f''(x) > 0$. Hence this last function does have an inflection point at $x = 1$. However, it does not have a local minimum at $x = -1$. Indeed, it has a local maximum at this point, as for $x < -1$, we have $f'(x) > 0$ and for $= 1 < x < 3$, we have $f'(x) < 0$ which in turn by the First Derivative Test, f has a LOCAL MAX. at $x = -1$. Consequently NO VALUES of a and b make $f(x) = x^3 + ax^2 + bx$ have a local minimum at $x = -1$ and a point of inflection $x = 1$.

p.385, pr.88

5. Consider the function $y = \dfrac{x^2-4}{x^2-2}$. You may assume that $y' = \dfrac{4x}{(x^2-2)^2}$ and $y'' = -\dfrac{12x^2+8}{(x^2-2)^3}$. Use this information to graph the function.

(a) $\boxed{\text{4 Points}}$ Give the asymptotes.

Solution: There are two vertical asymptotes: $x = \pm\sqrt{2}$, as the limits $\lim\limits_{x\to\sqrt{2}^+} \dfrac{x^2-4}{x^2-2} = -\infty$ and $\lim\limits_{x\to\sqrt{2}^-} \dfrac{x^2-4}{x^2-2} = +\infty$ $\lim\limits_{x\to-\sqrt{2}^+} \dfrac{x^2-4}{x^2-2} = -\infty$ $\lim\limits_{x\to-\sqrt{2}^-} \dfrac{x^2-4}{x^2-2} = -\infty$ indicate this is the case. Next $y = 1$ is the horizontal asymptote for the graph, as

$$\lim_{x\to\pm\infty} \frac{x^2-4}{x^2-2} = \lim_{x\to\pm\infty} \frac{1-4/x^2}{1-2/x^2} = 1$$

Hence $y = 1$ is the horizontal asymptote.

p.105, pr.25

(b) $\boxed{\text{4 Points}}$ Find the points where maximum and minimum values occur.

Solution: Since $y' = \dfrac{4x}{(x^2-2)^2} = 0 \Rightarrow x = 0$ is the only critical point, graph is decreasing on $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, 0)$ and increasing on $(0, \sqrt{2}) \cup (\sqrt{2}, \infty)$. Hence there is a local minimum at the point $(0, 2)$. There is no local maximum value.

p.147, pr.44

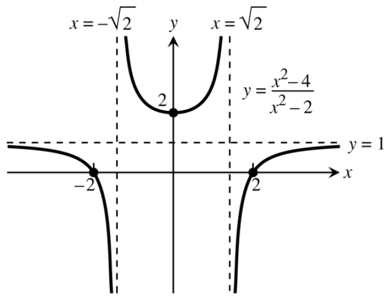
(c) $\boxed{\text{4 Points}}$ Give the inflection points.

Solution: Notice that the graph has no point of inflection as $y'' = -\dfrac{12x^2+8}{(x^2-2)^3} \neq 0$ for any x in the domain.

p.147, pr.44

(d) $\boxed{\text{8 Points}}$ *Sketch a graph* of the function. Label the asymptotes, critical points and the inflection points.

Solution:



p.211, pr.44