Cep telefonunuzu gözetmene teslim ediniz / Deposit your cell phones to invigilator

Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place a box around your answer to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 80 min.

Do not write in the table to the right.

Problem	Points	Score
1	24	
2	25	
3	27	
4	24	
Total:	100	

1. Suppose that $g(\theta) = \dfrac{5\cos\theta}{4\theta - 2\pi}$.

   (a) 12 Points If it exists, find the limit $\lim_{\theta\to\pi/2} g(\theta)$.

   Solution: The limit leads to the indeterminate $\frac{0}{0}$. Hence using L'Hôpital's Rule, we have

   $$\lim_{\theta\to 0}\frac{5\cos\theta}{4\theta-2\pi} \underset{\text{L'H}}{\overset{[\frac{0}{0}]}{=}} \lim_{\theta\to 0}\frac{-5\sin\theta}{4} = \frac{-5\sin(\pi/2)}{4} = \boxed{-\frac{5}{4}}$$

   p.652, pr.3

   (b) 12 Points Find the value $g(\pi/2)$ if $g(\theta)$ is continuous everywhere.

   Solution: $g(\theta)$ is continuous at $t=\pi/2$ iff $g(\pi/2) = \lim_{\theta\to\pi/2}$. That is, the value must be $\boxed{g(\pi/2) = -\frac{5}{4}}$.

   p.652, pr.3

2. (a) 12 Points Find the value of dy/dt at $t = 0$ if $y = 3\sin(2x)$ and $x = t^2 + \pi$.

Solution:

$$x = t^2 + \pi \Rightarrow \frac{dx}{dt} = \frac{d}{dt}(t^2 + \pi) = 2t;$$

$$y = 3\sin(2x) \Rightarrow \frac{dy}{dx} = \frac{d}{dx}(3\sin(2x)) = 3\sin(2x)\left[\frac{d}{dx}(2x)\right] = 3\cos(2x) \bullet 2$$

$$= 6\cos(2x) = 6\cos(2(t^2 + \pi)) = 6\cos(2t^2 + 2\pi) = 6\cos(2t^2)$$

$$\Rightarrow \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = \left[6\cos(2t^2)\right][2t] \Rightarrow \left[\frac{dy}{dt}\right]_{t=0} = 6\cos(0) \bullet (0) = \boxed{0}.$$

p.192, pr.57

(b) 13 Points Let $f(x) = 3x - x^3$. Show that the equation $f(x) = -4$ has a solution in the interval $[2,3]$.

Solution: Let $g(x) = f(x) + 4 = 3x - x^3 + 4$ where $x \in [2,3]$. Because g is a polynomial, it is continuous everywhere. Moreover,

- $g(2) = 2 > 0$
- $g(3) = -14 < 0$
- g is continuous on$[2,3]$

Hence by the Intermediate Value Theorem,

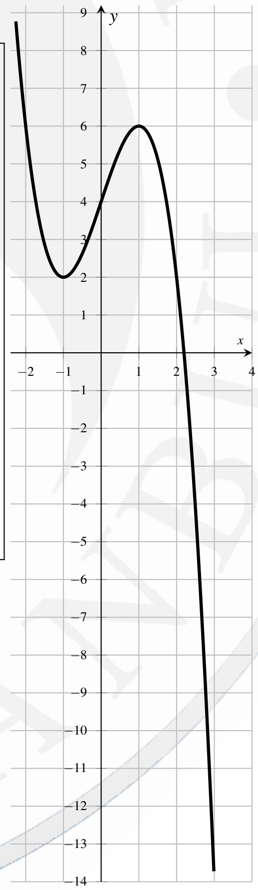
$$g(x) = 0$$

has a root on [2,3]. Therefore

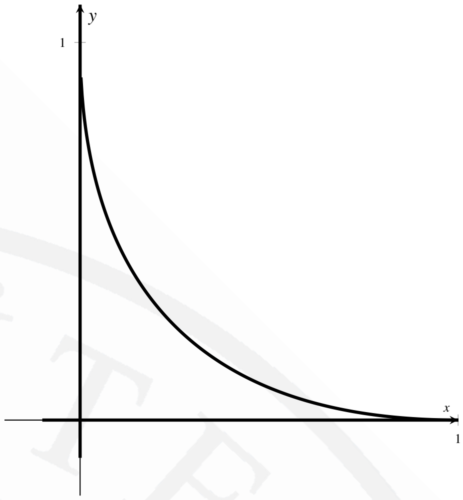
$$f(x) = -4$$

has a solution in the interval $[2,3]$.

4.4, pr.102



3. (a) 13 Points Find the total area of the region cut from the first quadrant by the curve $x^{1/2} + y^{1/2} = a^{1/2}$.



Solution:

$$\begin{aligned} A &= \int_0^a (a^{1/2} - x^{1/2})^2 \, dx = \int_0^a (a - 2\sqrt{a}x^{1/2} + x) \, dx \\ &= \left[ax - \frac{4}{3}\sqrt{a}x^{3/2} + \frac{x^2}{2} \right]_0^a \\ &= aa - \frac{4}{3}\sqrt{a}a\sqrt{a} + \frac{a^2}{2} \\ &= a^2\left(1 - \frac{4}{3} + \frac{1}{2}\right) = \frac{a^2}{6}(6 - 8 + 3) = \frac{a^2}{6} \end{aligned}$$

p.879, pr.42

(b) 14 Points Evaluate the integral $\displaystyle\int_{-\pi/3}^{\pi/3} 12\cos^2(4x)\sin(4x) \, dx$.

Solution: Let $u = \cos(4x)$. Then $du = -4\sin(4x) \ dx$. When $x = \pm\pi/3$, we have $u = \cos(\pm 4\pi/3) = -1/2$. Hence

$$\begin{aligned} \int_{-\pi/3}^{\pi/3} 12\cos^2(4x)\sin(4x) \, dx &= -3\int_{-\pi/3}^{\pi/3} \underbrace{(\cos(4x))^2}_{u^2} (-4\sin(4x)) \ dx_{du} \\ &= -3\int_{-1/2}^{-1/2} u^2 \, du = \boxed{0} \end{aligned}$$

p.112, pr.26

4. (a) 12 Points Evaluate the integral $\int_1^8 \frac{\log_4 \theta}{\theta}\, d\theta$.

Solution: First notice that

$$\frac{\log_4 \theta}{\theta} = \frac{\frac{\ln\theta}{\ln 4}}{\theta} = \frac{\ln\theta}{\theta \ln 4} = \frac{1}{\ln 4}\frac{\ln\theta}{\theta}.$$

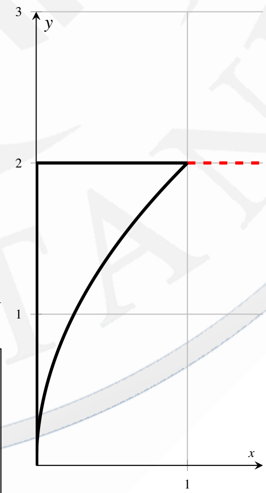
Now let $u = \ln\theta$. Then $du = \frac{1}{\theta}\, d\theta$. When $\theta = 1$, we have $u = \ln 1 = 0$ and when $\theta = 8$, we have $u = \ln 8 = 3\ln 2$. Therefore

$$\int_1^8 \frac{\log_4 \theta}{\theta}\, d\theta = \int_1^8 \frac{\ln\theta}{\ln 4}\frac{1}{\theta}\, d\theta = \frac{1}{\ln 4}\int_1^8 \ln\theta \frac{1}{\theta}\, d\theta = \frac{1}{\ln 4}\int_0^{\ln 8} u\, du$$

$$= \frac{1}{\ln 4}\left[\frac{u^2}{2}\right]_0^{\ln 8} = \frac{1}{2\ln 4}(3\ln 2)^2 = \frac{9\ln 2 \cancel{\ln 2}}{4\cancel{\ln 2}} = \boxed{\frac{9}{4}\ln 2}$$

p.241, pr.45

(b) 12 Points Find the volume of the solid generated by revolving the region bounded by $y = 2\sqrt{x}$, $y = 2$, and $x = 0$ about x-axis.



Solution: If we use the method of washers, the inner radius is $r(x) = 2\sqrt{x}$ and the outer radius is $R(x) = 2$. Hence the volume of revolution is

$$V = \int_0^1 \pi\{[R(x)]^2 - [r(x)]^2\}\, dx = \pi\int_0^1 \{(2)^2 - (2\sqrt{x})^2\}\, dx$$

$$= \pi\int_0^1 \{4 - 4x\}\, dx = \pi\left[4x - 2x^2\right]_0^1$$

$$= 4\pi\left(1 - \frac{1}{2}\right) = \boxed{2\pi}$$

p.212, pr.85