



Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

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Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives.**
- Place a box around your answer to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 90 min.

Do not write in the table to the right.

Problem	Points	Score
1	30	
2	12	
3	33	
4	25	
Total:	100	

1. (a) **8 Points** Find the value of $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right)$.

- (b) **12 Points** Find the integral $\int_1^2 \frac{8 dx}{x^2 - 2x + 2}$.

Solution:

$$\begin{aligned}\int_1^2 \frac{8 dx}{x^2 - 2x + 2} &= 8 \int_1^2 \frac{dx}{1 + (x^2 - 2x + 1)} = 8 \int_1^2 \frac{dx}{1 + (x-1)^2} \\ &= 8 \left[\tan^{-1}(x-1) \right]_1^2 \\ &= 8 \left[\tan^{-1}(2-1) - \tan^{-1}(1-1) \right] = 8 \left(\frac{\pi}{4} - 0 \right) = \boxed{2\pi}\end{aligned}$$

p.115, pr.11

- (c) **10 Points** Find the derivative $\frac{dy}{dx}$ if $y = \tan^{-1} \sqrt{x^2 - 1} + \csc^{-1} x$ where $x > 1$.

Solution:

$$\begin{aligned}y &= \tan^{-1} \sqrt{x^2 - 1} + \csc^{-1} x = \tan^{-1} (x^2 - 1)^{1/2} + \csc^{-1} x \\ \frac{dy}{dx} &= \frac{\frac{1}{2}(x^2 - 1)^{-1/2} 2x}{1 + ((x^2 - 1)^{1/2})^2} - \frac{1}{|x|\sqrt{x^2 - 1}} = \frac{\frac{x}{\sqrt{x^2 - 1}}}{x^2} - \frac{1}{|x|\sqrt{x^2 - 1}} = \frac{1}{x\sqrt{x^2 - 1}} - \frac{1}{|x|\sqrt{x^2 - 1}} = \boxed{0} \text{ as } x > 1\end{aligned}$$

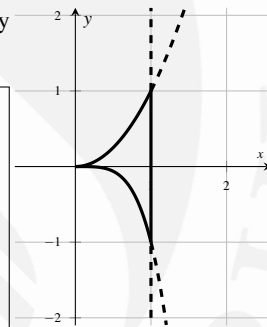
p.192, pr.87

2. **12 Points** Find the volume of the solid generated by revolving the region bounded by $y = x^2$, $y = -x^4$ and $x = 1$ about y-axis.

Solution: If we use the shells, we slice vertically and so we have

$$\begin{aligned}V &= \int_a^b 2\pi \left(\begin{array}{c} \text{shell} \\ \text{radius} \end{array} \right) \left(\begin{array}{c} \text{shell} \\ \text{height} \end{array} \right) dx \\ &= \int_0^1 2\pi x (x^2 - (-x^4)) dx \\ &= 2\pi \int_0^1 (x^3 + x^5) dx = 2\pi \left[\frac{x^4}{4} + \frac{x^6}{6} \right]_0^1 \\ &= 2\pi \left(\frac{1}{4} + \frac{1}{6} \right) = \boxed{\frac{5\pi}{6}}.\end{aligned}$$

p.191, pr.22



3. (a) 11 Points Find $\frac{dy}{dx}$ if $y = \int_{\sec x}^2 \frac{1}{t^2 + 1} dt$.

Solution: First let $u = \sec x$ and so $du = \sec x \tan x dx$. Now we have

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \left(\int_{\sec x}^2 \frac{1}{1+t^2} dt \right) = \frac{d}{dx} \left(- \int_2^{\sec x} \frac{1}{1+t^2} dt \right) = - \frac{d}{dx} \left(\int_2^u \frac{1}{1+t^2} dt \right) = - \frac{d}{du} \left(\int_2^u \frac{1}{1+t^2} dt \right) \frac{du}{dx} \\ &= - \frac{1}{1+u^2} \sec x \tan x \\ &= - \frac{1}{1+\sec^2 x} \sec x \tan x = - \frac{\sec x \tan x}{1+\sec^2 x} \end{aligned}$$

p.191, pr.22

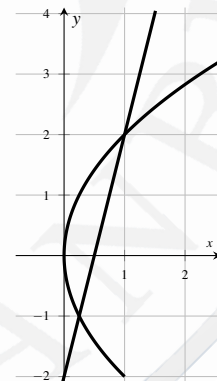
- (b) 10 Points Evaluate the integral $\int_0^{\pi/2} \frac{5 \sin x \cos x}{\sqrt{4+5 \cos^2 x}} dx$.

Solution: Let $u = 4 + 5 \cos^2 x$ and so $du = -10 \cos x \sin x dx$. When $x = 0$, we have $u = 9$ and when $x = \pi/2$, we have $u = 4$. Therefore

$$\begin{aligned} \int_0^{\pi/2} \frac{5 \sin x \cos x}{\sqrt{4+5 \cos^2 x}} dx &= - \frac{5}{10} \int_0^{\pi/2} \frac{1}{\sqrt{4+5 \cos^2 x}} (-10) \sin x \cos x dx \\ &= - \frac{1}{2} \int_9^4 \frac{1}{\sqrt{u}} du \\ &= \int_4^9 \frac{1}{2\sqrt{u}} du = \left[\sqrt{u} \right]_4^9 = \sqrt{9} - \sqrt{4} = 3 - 2 = \boxed{1} \end{aligned}$$

p.94, pr.10

- (c) 12 Points Find the area of the region enclosed by $y^2 = 4x$ and $y = 4x - 2$.



Solution: Let us find the intersection points:

$$\frac{y^2}{4} = \frac{y+2}{4} \Rightarrow y^2 - y - 2 = 0 \Rightarrow (y-2)(y+1) = 0 \Rightarrow y = -1 \text{ or } y = 2 \Rightarrow c = -1, d = 2;$$

$$f(y) = \frac{y+2}{4} \text{ and } g(y) = \frac{y^2}{4} \text{ and so}$$

$$\begin{aligned} \text{AREA} &= \int_c^d [f(y) - g(y)] dy = \int_{-1}^2 \left(\frac{y+2}{4} - \frac{y^2}{4} \right) dy = \frac{1}{4} \int_{-1}^2 (y+2-y^2) dy = \frac{1}{4} \left[\frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_{-1}^2 \\ &= \frac{1}{4} \left[\left(\frac{4}{2} + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} - 2 + \frac{1}{3} \right) \right] = \boxed{\frac{9}{8}} \end{aligned}$$

p.321, pr.21

4. (a) **11 Points** For what value(s) of k will the curve $y = x^3 + kx^2 + 3x - 4$ have *exactly one* horizontal tangent?

Solution: $f'(x) = 3x^2 + 2kx + 3 = 0$ (slope must be zero, for horizontal tangent)

To determine the types and number of roots of a quadratic equation $ax^2 + bx + c = 0$, we must examine the discriminant $\Delta = b^2 - 4ac$:

- If $b^2 - 4ac > 0$ then there are two real (unique) roots
- If $b^2 - 4ac = 0$ then there is one (double) real root
- If $b^2 - 4ac < 0$ then there are two imaginary roots

For single tangent, $b^2 - 4ac = 0$,

$$\text{So, } 4k^2 - 36 = 0$$

$$k^2 - 9 = 0$$

$$\Rightarrow k = \pm 3.$$

p.257, pr.4

- (b) **14 Points** Sketch the graph of $y = \frac{2x}{x+5} = 2 - \frac{10}{x+5}$. Find the asymptotes. Find all local maximum/minimum points and inflection points, if any. You may assume $y' = \frac{10}{(x+5)^2}$ and $y'' = -\frac{20}{(x+5)^3}$.

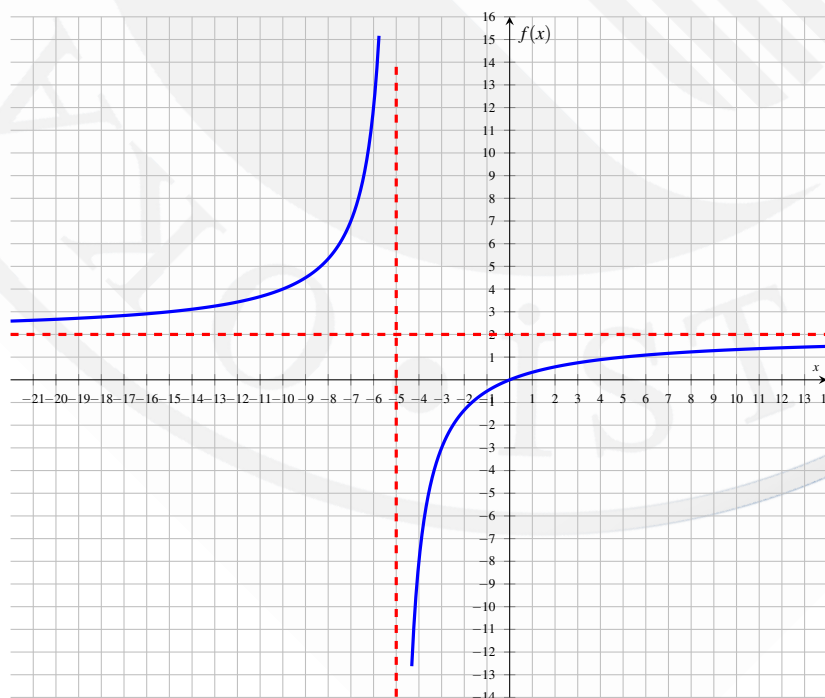
Solution: First $\lim_{x \rightarrow -5^+} \frac{2x}{x+5} = -\infty$, $\lim_{x \rightarrow -5^-} \frac{2x}{x+5} = +\infty$. So the line $x = -5$ is a vertical asymptote. Since $\lim_{x \rightarrow \pm\infty} \frac{2x}{x+5} = 2$, the line $y = 2$ is a horizontal asymptote.

Since $y' = \frac{10}{(x+5)^2} > 0$ for all $x \neq -5$, the graph is increasing on $(-\infty, -5) \cup (-5, \infty)$. Therefore no local maxima or minima occurs.

Moreover, we have $y'' = \frac{20}{(x+5)^3}$ and so

$$y'' \begin{cases} > 0, & \text{on } (-\infty, -5) & \text{y is concave up} \\ < 0, & \text{on } (-5, \infty) & \text{y is concave down} \end{cases}$$

Hence f is concave up on $(-\infty, -5)$ and concave down on $(-5, \infty)$. Also graph has *no point of inflection* as there is no tangent line at $x = -5$.



p.212, pr.85