Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place a box around your answer to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 70 min.

Do not write in the table to the right.

Problem	Points	Score
1	33	
2	31	
3	36	
Total:	100	

1. (a) $\boxed{\text{10 Points}}$ Find the integral $\int_0^{\pi} 5(5-4\cos t)^{1/4} \sin t \, dt$.

Solution: Let $u = 5-4\cos t$ and so $du = 4\sin t\, dt$. When $t = 0$, we have $u = 5-4\cos 0 = 1$ and when $t = \pi$, we have $u = 5-4\cos\pi = 9$. Hence

$$\int_0^{\pi} 5(5-4\cos t)^{1/4} \sin t \, dt = \frac{5}{4}\int_0^{\pi} (5-4\cos t)^{1/4} 4\sin t \, dt$$
$$= \frac{5}{4}\int_1^9 u^{1/4}\, du = \frac{5}{4}\left[\frac{u^{5/4}}{5/4}\right]_1^9 = 9^{5/4} - 1^{5/4} = \boxed{3^{5/2}-1}$$

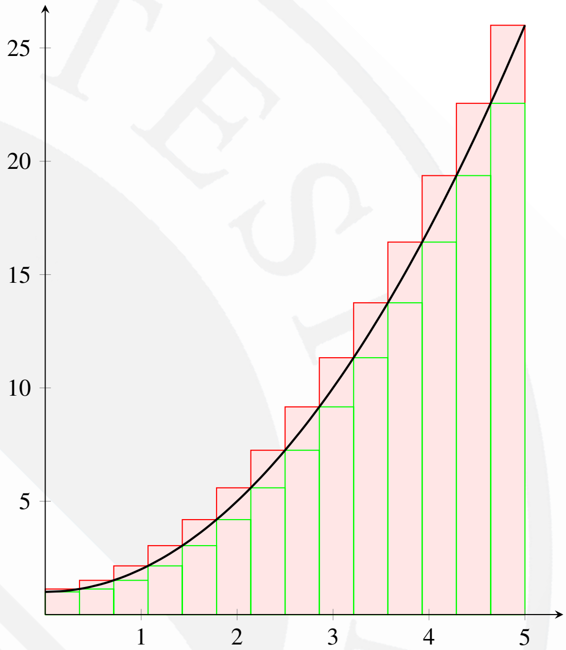
p.191, pr.22

(b) $\boxed{\text{13 Points}}$ Find the area under $f(x) = x^2+1$ over $[0,5]$ as follows. First, find a formula, in terms of n, for the Riemann sum

$$S_n = \sum_{k=1}^{n} f(c_k)\,\Delta x$$

by dividing $[0,5]$ into n equal subintervals, and using $c_k = x_k$, the right-hand endpoint of $[x_{k-1}, x_k]$ for each k. Then find $\lim_{n\to\infty} S_n$.

Hint: You'll need $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.



Solution: Let $\Delta x = \frac{b-a}{n} = \frac{5-0}{n} = \frac{5}{n}$ and $c_k = k\Delta x = \frac{5k}{n}$. The right-hand sum is

$$\sum_{k=1}^{n} f(c_k)\,\Delta x = \sum_{k=1}^{n}\left(c_k^2+1\right)\frac{5}{n} = \sum_{k=1}^{n}\left(\left(\frac{5k}{n}\right)^2+1\right)\frac{5}{n} = \frac{5}{n}\sum_{k=1}^{n}\left(\frac{25k^2}{n^2}+1\right)$$
$$= \frac{125}{n^3}\sum_{k=1}^{n} k^2 + \frac{5}{n}\sum_{k=1}^{n}(1)$$
$$= \frac{125}{n^3}\frac{n(n+1)(2n+1)}{6} + \left(\frac{5}{n}\right)(n) = \frac{125}{6}\frac{n}{n}\frac{n+1}{n}\frac{2n+1}{n} + 5 = \frac{125}{6}(1)\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)+5.$$

Hence the area is

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left(\frac{125}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)+5\right) = \frac{125}{6}(1+0)(2+0)+5 = \frac{125}{6}(1)(2)+5 = \boxed{\frac{140}{3}}$$

p.192, pr.87

(c) $\boxed{\text{10 Points}}$ Find $\frac{dy}{dx}$ if $y = \int_{\sqrt{x}}^{0} \sin(t^2)\, dt$.

Solution: First, notice that $y = \int_{\sqrt{x}}^{0} \sin(t^2)\, dt = -\int_0^{\sqrt{x}} \sin(t^2)\, dt$. Then

$$\frac{dy}{dx} = \frac{d}{dx}\left(-\int_0^{\sqrt{x}} \sin(t^2)\, dt\right) = -\frac{d}{dx}\left(\int_0^{\sqrt{x}} \sin(t^2)\, dt\right)$$

Now let $u = \sqrt{x}$. Then $\dfrac{du}{dx} = \dfrac{1}{2\sqrt{x}}$. By the Chain Rule, we have

$$\frac{dy}{dx} = -\frac{d}{dx}\int_0^u \sin(t^2)\,\mathrm{d}t = -\frac{d}{du}\left(\int_0^u \sin(t^2)\,\mathrm{d}t\right)\frac{du}{dx} = -\sin(u^2)\left(\frac{1}{2\sqrt{x}}\right)$$
$$= \boxed{-\frac{\sin x}{2\sqrt{x}}}$$

p.192, pr.57

2. Suppose $y = \dfrac{x+1}{x-3} = 1 + \dfrac{4}{x-3}$. You may assume that $y' = -\dfrac{4}{(x-3)^2}$ and $y'' = \dfrac{8}{(x-3)^3}$.

(a) $\boxed{\text{11 Points}}$ Give the *asymptotes*.

Solution: First $\lim\limits_{x\to 3^+} \dfrac{x+1}{x-3} = +\infty$, $\lim\limits_{x\to 3^-} \dfrac{x+1}{x-3} = -\infty$. So the line $x = 3$ *is a vertical asymptote*. Since $\lim\limits_{x\to\pm\infty} \dfrac{x+1}{x-3} = 1$, the line $y = 1$ is a horizontal asymptote.

p.257, pr.4

(b) $\boxed{\text{10 Points}}$ Find the *intervals* where the graph is *increasing* and *decreasing*. Find the *local maximum* and *minimum* values. Determine the concavity and the points of inflection.

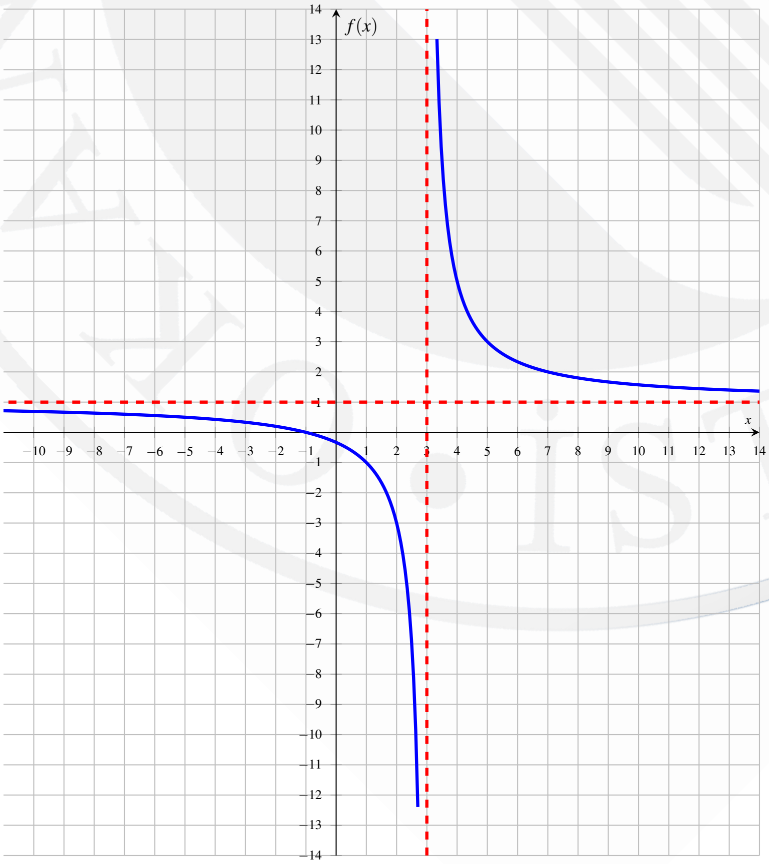
Solution: Since $y' = -\dfrac{4}{(x-3)^2} < 0$ for all $x \neq 3$, the graph is decreasing on $(-\infty, 3) \cup (3, \infty)$. Therefore no local maxima or minima occurs.

Moreover, we have $y'' = \dfrac{8}{(x-3)^3}$ and so

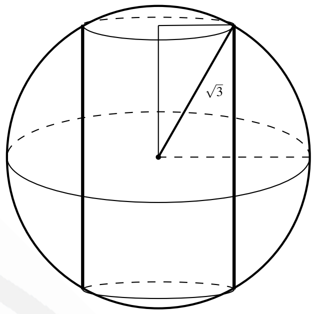
$$y'' \begin{cases} > 0, & \text{on } (3,\infty) \quad \text{y is concave up} \\ < 0, & \text{on } (-\infty,3) \quad \text{y is concave down} \end{cases}$$

Hence f is concave up on $(3,\infty)$ and concave down on $(-\infty,3)$. Also graph has *no point of inflection* as there is no tangent line at $x = 3$.

(c) $\boxed{\text{10 Points}}$ *Sketch the graph* of the function. Label the asymptotes, critical points and the inflection points.



3. (a) $\boxed{\text{10 Points}}$ Find the height and radius of the largest right circular cylinder that can be put in a sphere of radius of $\sqrt{3}$.



Solution: Let r and h denote the radius and height of the cylinder respectively. Then, using trigonometry (right triangles - draw a line from the center to where the line touches the circle), you can find h in terms of r. From the diagram, we have $\left(\frac{h}{2}\right)^2 + r^2 = (\sqrt{3})^2 \Rightarrow r^2 = \frac{12-h^2}{4}$. The volume of the cylinder is

$$V = \pi r^2 h = \pi\left(\frac{12-h^2}{4}\right)h = \frac{\pi}{4}\left(12h - h^3\right), \quad \text{where } 0 \le h \le 2\sqrt{3}.$$

Then $V'(h) = \frac{3\pi}{4}(2+h)(2-h) \Rightarrow$ the critical points are -2 and 2, but $-2 \notin [0, 2\sqrt{3}]$. At $h=2$, there is a maximum since $V''(h) = -\frac{3\pi}{2}h$ is $V''(2) = -3\pi < 0$ a negative value. The dimensions of the largest cylinder are radius $= \boxed{\sqrt{2}}$ and height $= \boxed{2}$.

p.80, pr.23

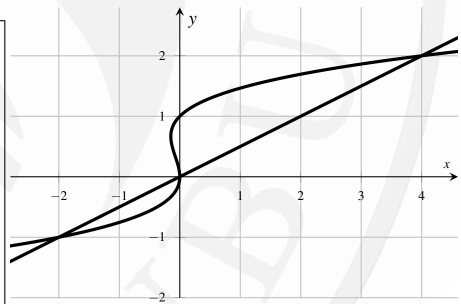
(b) $\boxed{\text{12 Points}}$ Find the integral $\int x^4(1-x^5)^{-1/5}\,\mathrm{d}x$.

Solution: Let $u = 1 - x^5$ and so $du = -5x^4$. Then

$$\begin{aligned}\int x^4(1-x^5)^{-1/5}\,\mathrm{d}x &= -\frac{1}{5}\int (1-x^5)^{-1/5}(-5)x^4\,\mathrm{d}x \\ &= -\frac{1}{5}\int u^{-1/5}\,\mathrm{d}u = -\frac{1}{5}\frac{u^{-1/5+1}}{-1/5+1} + C = -\frac{1}{5}\frac{5}{4}u^{4/5} + C \\ &= \boxed{-\frac{1}{4}(1-x^5)^{4/5} + C}\end{aligned}$$

p.115, pr.11

(c) $\boxed{\text{14 Points}}$ From the figure, find the total area enclosed by $x = y^3 - y^2$ and $x = 2y$.



Solution: For $y \in [-1, 0]$, we have $y^3 - y^2 \ge 2y$ and for $y \in [0, 2]$, we have the reverse inequality $2y \ge y^3 - y^2$. Therefore

$$\begin{aligned}\text{TOTAL AREA} &= \int_{-1}^{0}(y^3 - y^2 - 2y)\ \mathrm{d}y + \int_{0}^{2}(2y - y^3 + y^2)\ \mathrm{d}y \\ &= \left[\frac{1}{4}y^4 - \frac{1}{3}y^3 - y^2\right]_{-1}^{0} + \left[y^2 - \frac{1}{4}y^4 + \frac{1}{3}y^3\right]_{0}^{2} \\ &= 0 - \frac{1}{4} - \frac{1}{3} + 1 + \left(4 - 4 + \frac{8}{3}\right) \\ &- 0 \\ &= \frac{5}{2} + \frac{8}{3} = \boxed{\frac{37}{12}}\end{aligned}$$

p.94, pr.10