Math 113 Summer 2018 **First Exam** July 10, 2018

Your Name / Ad - Soyad (**75 min.**) Signature / İmza

Student ID # / Öğrenci No (**use a blue pen!**)

Problem	1	2	3	4	Total
Points:	30	25	20	25	100
Score:					

Time limit is **75 minutes**. If you need more room on your exam paper, you may use the empty spaces on the paper. You have to show your. Answers without reasonable-even if your results are true- work will either get zero or very little credit.

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1. (a) (10 Points) Find the limit $\lim_{y\to -2} \frac{y+2}{y^2+5y+6}$.

You are not allowed to use L'Hôpital's rule.

Solution: We have

$$\lim_{y\to -2} \frac{y+2}{y^2+5y+6} = \lim_{y\to -2} \frac{\cancel{(y+2)}^{1}}{\cancel{(y+2)}^{1}(y+3)}$$
$$= \lim_{y\to -2} \frac{1}{y+3} = \frac{1}{-2+3} = \boxed{1}$$

p.72, pr.15

(b) (10 Points) Find the second derivative y'' if $y = \frac{(x^2+x)(x^2-x+1)}{x^4}$.

Solution:

$$y = \frac{(x^2+x)(x^2-x+1)}{x^4} = \frac{x^4+x}{x^2} = 1 + \frac{1}{x^3} \Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(1+x^{-3}\right) = 0 - 3x^{-4} = \boxed{-\frac{3}{x^4}}$$
$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(-\frac{3}{x^4}\right) = \frac{d}{dx}\left(-3x^{-4}\right) = (-3)(-4)x^{-4-1} = \boxed{\frac{12}{x^5}}$$

p.94, pr.10

(c) (10 Points) Suppose $y = \tan^2(\sec(3t))$. Find the derivative $\frac{dy}{dt}$

Solution: We have by the Chain Rule,

$$\frac{dy}{dt} = \frac{d}{dt}(\tan(\sec(3t)))^2 = 2\tan(\sec(3t))\frac{d}{dt}(\tan(\sec(3t))) = 2\tan(\sec(3t))\sec^2(\sec(3t))\frac{d}{dt}(\sec(3t))$$
$$= 2\tan(\sec(3t))\sec^2(\sec(3t))\sec(3t)\tan(3t)\underbrace{\frac{d}{dt}(3t)}_{3} = \boxed{6\tan(\sec(3t))\sec^2(\sec(3t))\sec(3t)\tan(3t)}$$

p.94, pr.34

2. (a) (12 Points) Suppose $f(x) = 3-2x$, $L = -3$, $x_0 = 3$, $\varepsilon = 0.02$. Find an open interval about x_0 on which the inequality $|f(x)-L| < \varepsilon$ holds. Then, using the given information, give a value for $\delta > 0$ such that for all x satisfying $0 < |x-x_0| < \delta$ the inequality $|f(x)-L| < \varepsilon$ holds.

Solution: For the required interval, we want $|(3-2x)-3| < 0.02$. In other words, we need $|6-2x| < 0.02$. This is the same as writing $-0.02 < 6-2x < 0.02$. Solving this for x, we get

$$-6-0.02 < -2x < -6+0.02 \Rightarrow -6.02 < -2x < -5.98$$

and hence by dividing by -2 gives

$$2.96 < x < 3.01$$

that is an interval we want is then

$$\boxed{x \in (2.96, 3.01).}$$

For the second part, we want to find a $\delta > 0$ such that

$$\begin{aligned}|x-3| < \delta &\Rightarrow -\delta < x-3 < \delta\\ &\Rightarrow -\delta+3 < x < \delta+3\\ &\Rightarrow x \in (-\delta+3, \delta+3)\end{aligned}$$

Since $x \in (-2.01, -1.99)$, we can have $-\delta+3 = 2.96 \Rightarrow \delta = 0.04$ and $\delta+3 = 3.01 \Rightarrow \delta = 0.01$ and so solving these we can have $\delta = \min\{0.04, 0.01\} \Rightarrow \boxed{\delta = 0.01}$.

p.72, pr.8

(b) (13 Points) Use the formula $f'(x) = \lim_{z\to x} \frac{f(z)-f(x)}{z-x}$ to find an equation for the line tangent to the curve $y = \frac{8}{\sqrt{x}}$ at point $(4,4)$.

Solution: First the slope for the line at $(4,4)$ is

$$\begin{aligned}m &= \lim_{z\to 4} \frac{f(z)-f(4)}{z-4} = \lim_{z\to 4} \frac{\frac{8}{\sqrt{z}} - \frac{8}{\sqrt{4}}}{z-4}\\ &= \lim_{z\to 4} \frac{16-8\sqrt{z}}{(z-4)2\sqrt{z}} = \lim_{z\to 4} \frac{-8(\sqrt{z}-2)}{(\sqrt{z}-2)(\sqrt{z}+2)2\sqrt{z}}\\ &= \lim_{z\to 4} \frac{-8}{(\sqrt{z}+2)2\sqrt{z}} = \frac{-8}{(\sqrt{4}+2)2\sqrt{4}} = -\frac{8}{16} = -\frac{1}{2}\end{aligned}$$

Then the equation for the tangent is $y-4 = -\frac{1}{2}(x-4) \Rightarrow$ $\boxed{x+2y = 12}$.

p.72, pr.8

3. (a) (10 Points) Find $\frac{dy}{dx}$ if $2xy + y^2 = x + y$ defines y implicitly as a function of x.

Solution: We have

$$\begin{aligned} \frac{d}{dx}\left(2xy + y^2\right) &= \frac{d}{dx}(x+y) \\ \frac{d}{dx}(2xy) + \frac{d}{dx}(y^2) &= \frac{d}{dx}(x) + \frac{d}{dx}(y) \\ 2y + 2x\frac{dy}{dx} + 2y\frac{dy}{dx} - \frac{dy}{dx} &= 1 \\ (2x + 2y - 1)\frac{dy}{dx} &= 1 - 2y \\ \frac{dy}{dx} &= \boxed{\frac{1-2y}{2x+2y-1}} \end{aligned}$$

p.83, pr.52

Use Implicit Differentiation.

(b) (10 Points) Redefine the function $g(x) = \frac{x^2-16}{x^2-3x-4}$ so that it is *continuous at* $x = 4$ (continuous extension).

Solution: First notice that

$$\lim_{x\to 4} \frac{x^2-16}{x^2-3x-4} = \lim_{x\to 4} \frac{(x-4)(x+4)}{(x-4)(x+1)} = \lim_{x\to 4} \frac{x+4}{x+1} = \frac{4+4}{4+1} = \frac{8}{5}$$

and

$$g(x) = \begin{cases} \frac{x^2-16}{x^2-3x-4} & x \neq 4 \\ \frac{8}{5} & x = 4 \end{cases}$$

the graph has *a removable discontinuity* at $\boxed{\text{x=4}}$.

p.83, pr.52

You are not allowed to use L'Hôpital's Rule.

4. (a) (15 Points) Find the domain and all the asymptotes and graph $y = \dfrac{8}{x^2-4}$.

Solution: First the domain is $(-\infty,-2) \cup (-2,2) \cup (2,\infty)$. Notice that

$$\lim_{x\to\pm\infty} \frac{8}{x^2-4} = \lim_{x\to\pm\infty} \frac{8/x^2}{1-4/x^2} = 0$$

and so $\boxed{y=0}$ *is the (only) horizontal asymptote*. Moreover, since

$$\lim_{x\to 2^+} \frac{8}{x^2-4} = \infty, \quad \lim_{x\to 2^-} \frac{8}{x^2-4} = -\infty$$

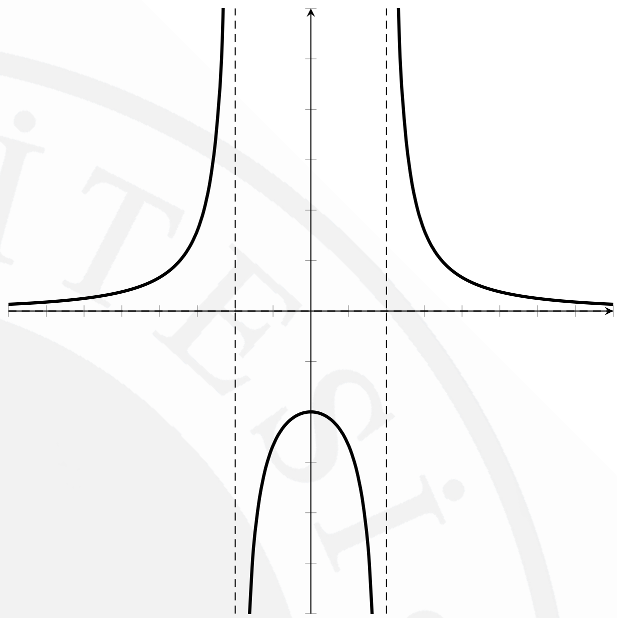
and

$$\lim_{x\to -2^+} \frac{8}{x^2-4} = -\infty, \quad \lim_{x\to -2^-} \frac{8}{x^2-4} = \infty$$

the graph has *two vertical asymptotes*, and they are $\boxed{\text{x=2}}$ and $\boxed{\text{x=-2}}$.

p.95, pr.68

$y = f(x) = \dfrac{8}{x^2-4}$



(b) (10 Points) Find the limit $\displaystyle\lim_{x\to\pi} \sin\left(\frac{x}{2} + \sin x\right)$.

Solution: By continuity of $f(x) = \sin x$,

$$\begin{aligned}\lim_{x\to\pi} \sin\left(\frac{x}{2} + \sin x\right) &= \sin\left(\lim_{x\to\pi}\left(\frac{x}{2} + \sin x\right)\right) \\ &= \sin\left(\left(\frac{\pi}{2} + \sin\pi\right)\right) = \sin\left(\frac{\pi}{2} + 0\right) = \boxed{1}\end{aligned}$$

p.112, pr.26