Cep telefonunuzu gözetmene teslim ediniz / Deposit your cell phones to invigilator

Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor' s Name / Öğretim Üyesi

Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 70 min.

Do not write in the table to the right.

Problem	Points	Score
1	40	
2	35	
3	25	
Total:	100	

1. (a) $\boxed{\text{15 Points}}$ Find the extreme value of the function $V(x) = x(10-2x)(16-2x)$, models the volume of a box, on $0 < x < 5$.

Solution:
lr V(x) $= 160\text{x-}52\text{x}^2 + 4x^3$
$V'(x) = 160 - 104x + 12x^2 = 4(40-26x+3x^2) = (3x-20)(x-2) = 0$ We find $x_1 = \dfrac{20}{3} \cong 6.6$ and $x_2 = 2$. Because $0 < x < 5$ $x = x_2 = 2$.That is, the extreme value of V in $(0,5)$ is $x = 2$.

(b) $\boxed{\text{10 Points}}$ Show that the function $g(t) = \sqrt{t} + \sqrt{1+t} - 4$ have exactly one **zero** in $(0,\infty)$.

Solution: $t = 0 \Rightarrow g(0) = \sqrt{0} + \sqrt{1} - 4 = -3$
$t = 15 \in (0,\infty) \Rightarrow \sqrt{t} + \sqrt{1+t} - 4 = \sqrt{15} + \sqrt{16} - 4 = \sqrt{15}$. By virtue of **mean value theorem**, the graph of the function g intersects the x-axis at least one point in $(0,\infty)$, because $g(0) < 0$ and for $t = 15 \in (0,\infty)$, $\sqrt{t} + \sqrt{1+t} - 4 = \sqrt{15} + \sqrt{16} - 4 = \sqrt{15} > 0$. That is, there is at least one root in this interval.
Now let's make sure that the root is the unique in $(0,\infty)$. For this, we find the derivative of the function: $g'(t) = \dfrac{1}{2\sqrt{t}} + \dfrac{1}{2\sqrt{1+t}}$.
This function is always positive(that is increasing) on $(0,\infty)$. Therefore, it is impossible to find an other root in this interval.

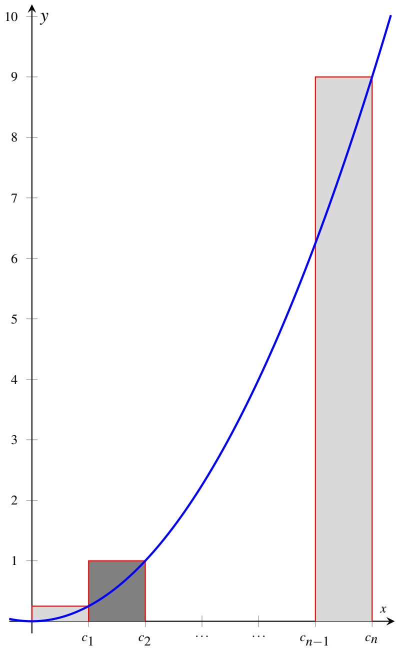
(c) 15 Points **(Minimizing Perimeter:)** What is the smallest perimeter possible for a rectangle whose area is $16cm^2$, and what are its dimensions?

Solution: lr Area: A=xy ⇒y=16$\frac{}{x}$
Perimeter:P= $2(x+y)$ ⇒Ç(x)=2(x+16$\frac{}{x)=\frac{2x^2+32}{x}}$

$Ç'(x) = \frac{x^2-16}{x^2} = 0 \Rightarrow Critical\ \ Points: x = \mp 4\ \ and\ \ x = 0$

Because the length of the sides can not be zero 0 the dimentions of the rectangle will be $x = 4$ and $y = \dfrac{16}{4} = 4$. $x = 4$ is local min because $Ç''(x) = \frac{32}{x^3} > 0$ for $x = 4$. So, the min value of the perimeter is $P(x) = 2(4+4) = 16cm$.

2. (a) 15 Points For the function $f(x) = x^2$, find a formula for the Riemann sum obtained by dividing the interval $[0,3]$ into n equal subintervals and using the right hand point for each c_k. Then take a limit of these sums as $n \to \infty$ to calculate the area under the curve over $[0,3]$.



Solution:

$$[a,b] = [0,3], \qquad \triangle x = \frac{b-a}{n} = \frac{3}{n}$$

$$c_k = x_k = a - \triangle x = 0 + k\frac{3}{n}$$

$$S_n = \sum_{k=1}^{n} f(x_k)\triangle x$$
$$= \sum_{k=1}^{n} ((\frac{3k}{n})^2)\frac{3}{n}$$
$$= \frac{27}{n^3}\sum_{k=1}^{n} k^2$$
$$= \frac{27}{n^3}(\frac{n(n+1)(2n+1)}{6})$$
$$= \frac{9(2n^3+3n+n)}{2n^3}$$

$$\text{Area} = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{9(2n^3+3n+n)}{n^3} = 9$$

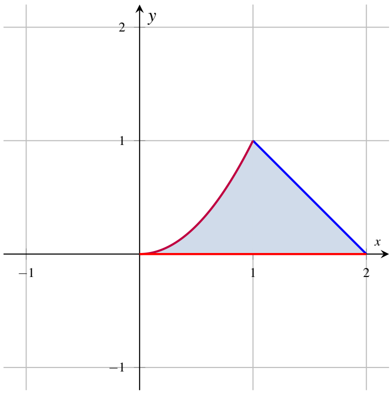
(b) $\boxed{\text{10 Points}}$ Use the substitution formula to evaluate the integral $\int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta$.

Solution:

$$\int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta$$
$$= -\frac{1}{2}\int_1^{\frac{1}{2}} \frac{1}{u^3} du$$
$$= -\frac{1}{2}\left(\frac{u^{-2}}{-2}\right)\Bigg|_1^{\frac{1}{2}}$$
$$= \frac{1}{4}\left((\frac{1}{2}^{-2}) - (1)^{-2}\right) = \frac{3}{4}$$

$$g(\theta) = u = \cos 2\theta$$
$$\frac{du}{d\theta} = -2\sin 2\theta$$
$$g(0) = \cos 0 = 1$$
$$g(\frac{\pi}{6}) = \cos\frac{\pi}{3} = \frac{1}{2}$$

(c) $\boxed{\text{10 Points}}$ Find the area of the "triangular"region bounded on the left by $x+y=2$, on the right by $y=x^2$, and below by $y=0$ (x-axis).

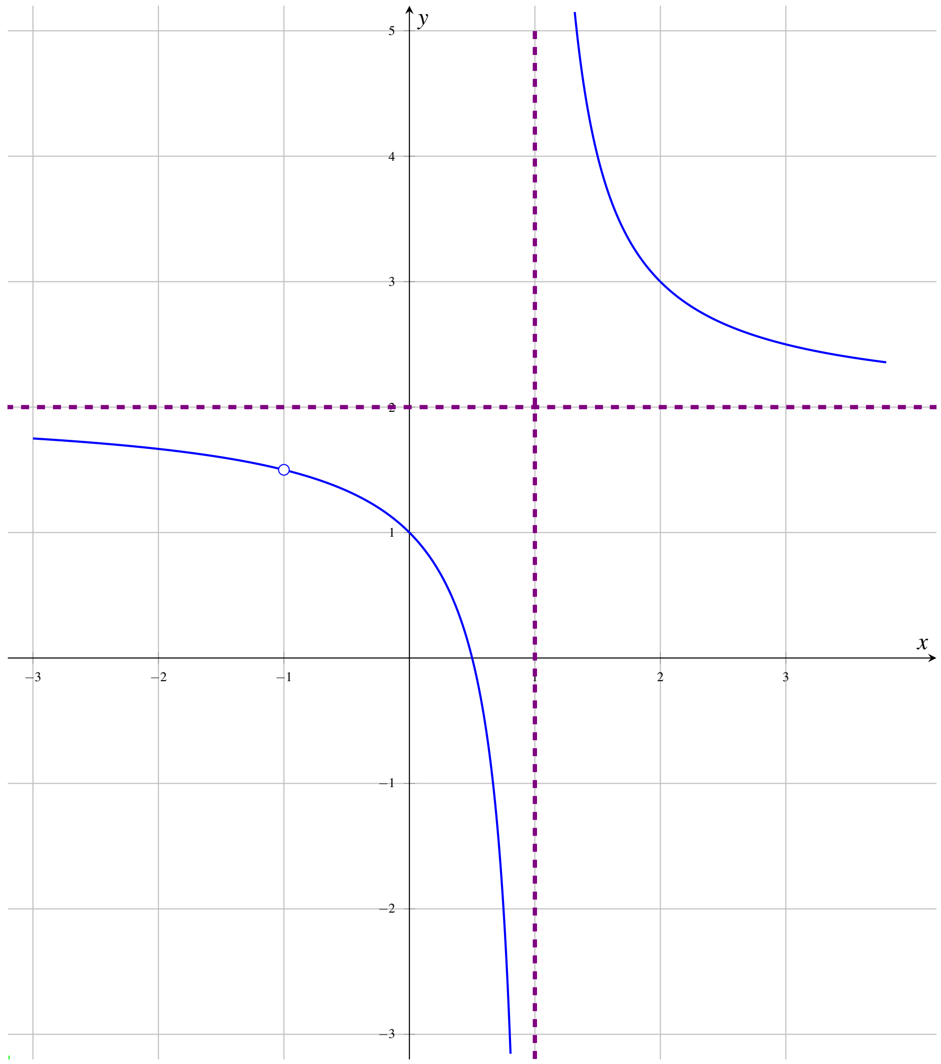


Solution:

$$y = 2 - x \quad \text{and} \quad y = x^2 \quad \text{intersect}$$
$$y = x^2 = 2 - x \Rightarrow (x-2)(x+1) = 0$$
$$x = 2 \Rightarrow y = 1$$
$$A = \int_0^1 x^2 dx + \int_1^2 (2-x)dx = \frac{x^3}{3}\Bigg|_0^1 + (2x - \frac{x^2}{2})\Bigg|_1^2$$
$$= \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

3. $\boxed{\text{25 Points}}$ Sketch the graph of the rational function $y = \dfrac{2x^2+x-1}{x^2-1}$. Please note that $y' = -\dfrac{1}{(x-1)^2}$ and $y'' = \dfrac{2}{(x-1)^3}$.

Solution:



Clearly y is undefined at $x=-1$ and $x=1$. We can reaarange y as
$$y = \frac{2x-1}{x-1} = 2 + \frac{1}{x-1}$$
. Beside this, y' is undefined at $x=1$ because $y' = -\frac{1}{(x-1)^2} = 0$. Therefore, the only critical point candidate is $x=1$, but it is not because y is undefined at $x=1$. We can also see that y'' is not defined at $x=1$. But there is no inflection point by virtue of the same reason.
Next we must find the asymptotes of the graph: $\lim_{x\to 1} y = \infty$. Hence $x=1$ is an vertical asymptote of the graph. Moreover $\lim_{x\to\infty} y = 2$.

So $y=2$ is a horizontal asymptote of the graph. There is not any obligue asymptote.
We can also say that

(i) f is increasing on the intervals $(-\infty,-1)$ and $(1,\infty)$;
(ii) f is decreasing on the intervals $(-\infty,1)$ and $(1,\infty)$;
(iii) f is concave down on the interval $(-\infty,1)$;
(iv) f is concave up on the interval $(1,\infty)$; and
(v) There is not any local maximum or local minimum.

The table below summarises this informations:

x	$-\infty$		1		∞
$f'(x)$		-		-	
$f''(x)$		-		+	
$f(x)$					