Cep telefonunuzu gözetmene teslim ediniz / Deposit your cell phones to invigilator

Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor' s Name / Öğretim Üyesi

Your Department / Bölüm

- Calculators, cell phones off and away!.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place a box around your answer to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 70 min.

Do not write in the table to the right.

Problem	Points	Score
1	40	
2	35	
3	25	
Total:	100	

1. (a) 15 Points Find the extreme value of the function $V(x) = x(10-2x)(16-2x)$, models the volume of a box, on $0 < x < 5$.

Solution:
lr V(x) $= 160x - 52x^2 + 4x^3$
$V'(x) = 160 - 104x + 12x^2 = 4(40 - 26x + 3x^2) = (3x-20)(x-2) = 0$ We find $x_1 = \frac{20}{3} \cong 6.6$ and $x_2 = 2$. Because $0 < x < 5$
$x = x_2 = 2$.That is, the extreme value of V in $(0,5)$ is $x = 2$.

(b) 10 Points Show that the function $g(t) = \sqrt{t} + \sqrt{1+t} - 4$ have exactly one **zero** in $(0,\infty)$.

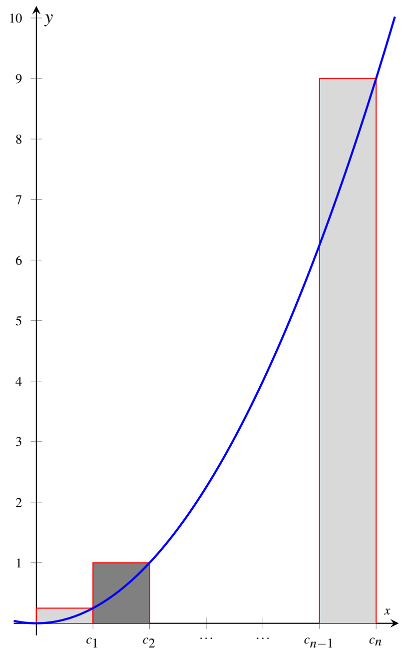
Solution: lr t=0 $\Rightarrow g(0) = \sqrt{0} + \sqrt{1} - 4 = -3$
$t \to \infty \Rightarrow g(t)|_{t\to\infty} = \lim_{t\to\infty} \sqrt{t} + \sqrt{1+t} - 4 = \infty$ By virtue of **mean value theorem**, the graph of the function g intersects the x-axis at least one point in $(0,\infty)$, because $g(0) < 0$ and $g(t)|_{t\to\infty} > 0$. That is, there is at least one root in this interval.
Now let's make sure that the root is the unique in $(0,\infty)$. For this, we find the derivative of the function: $g'(t) = \frac{1}{2\sqrt{t}} + \frac{1}{2\sqrt{1+t}}$.
This function is always positive(that is increasing) on $(0,\infty)$. Therefore, it is impossible to find an other root in this interval.

(c) 15 Points **(Minimizing Perimeter:)** What is the smallest perimeter possible for a rectangle whose area is $16cm^2$, and what are its dimensions?

Solution: lr Area: A=xy $\Rightarrow y = \frac{16}{x}$
$Perimeter: P = 2(x+y) \Rightarrow Ç(x) = 2(x + \frac{16}{x}) = \frac{2x^2+32}{x}$
$Ç'(x) = \frac{x^2-16}{x^2} = 0 \Rightarrow Critical\ \ Points: x = \mp 4\ \ and\ \ x = 0$

Because the length of the sides can not be zero0 the dimentions of the rectangle will be $x = 4$ and $y = \dfrac{16}{4} = 4$. So, the min value of the perimeter is $P(x) = 2(4+4) = 16cm$.

2. (a) 15 Points For the function $f(x) = x^2$, find a formula for the Riemann sum obtained by dividing the interval $[0,3]$ into n equal subintervals and using the right hand point for each c_k. Then take a limit of these sums as $n \to \infty$ to calculate the area under the curve over $[0,3]$.



Solution:

$$[a,b] = [0,3], \qquad \triangle x = \frac{b-a}{n} = \frac{3}{n}$$

$$c_k = x_k = a - \triangle x = 0 + k\frac{3}{n}$$

$$\begin{aligned} S_n &= \sum_{k=1}^{n} f(x_k)\triangle x \\ &= \sum_{k=1}^{n} \left(\left(\frac{3k}{n}\right)^2\right)\frac{3}{n} \\ &= \frac{27}{n^3}\sum_{k=1}^{n} k^2 \\ &= \frac{27}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) \\ &= \frac{9(2n^3+3n+n)}{2n^3} \end{aligned}$$

$$\text{Area} = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{9(2n^3+3n+n)}{n^3} = 9$$

(b) 10 Points Use the substitution formula to evaluate the integral $\int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta$.

Solution:

$$\int_0^{\frac{\pi}{6}} \cos^{-3} 2\theta \sin 2\theta d\theta$$

$$= -\frac{1}{2}\int_1^{\frac{1}{2}} \frac{1}{u^3} du$$

$$= -\frac{1}{2}\left(\frac{u^{-2}}{-2}\right)\Big|_1^{\frac{1}{2}}$$

$$= \frac{1}{4}\left((\frac{1}{2}^{-2}) - (1)^{-2}\right) = \frac{3}{4}$$

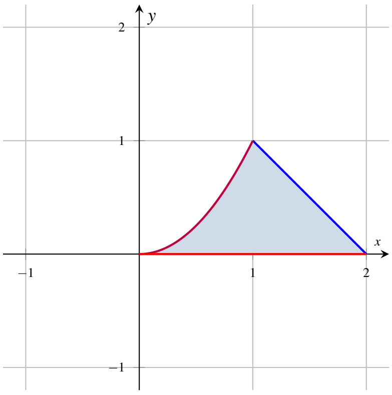
$$g(\theta) = u = \cos 2\theta$$

$$\frac{du}{d\theta} = -2\sin 2\theta$$

$$g(0) = \cos 0 = 1$$

$$g(\frac{\pi}{6}) = \cos\frac{\pi}{3} = \frac{1}{2}$$

(c) 10 Points Find the area of the "triangular" region bounded on the left by $x+y=2$, on the right by $y=x^2$, and below by $y=0$ (x-axis).



Solution:

$$y = 2-x \quad and \quad y = x^2 \quad intersect$$

$$y = x^2 = 2-x \Rightarrow (x-2)(x+1) = 0$$

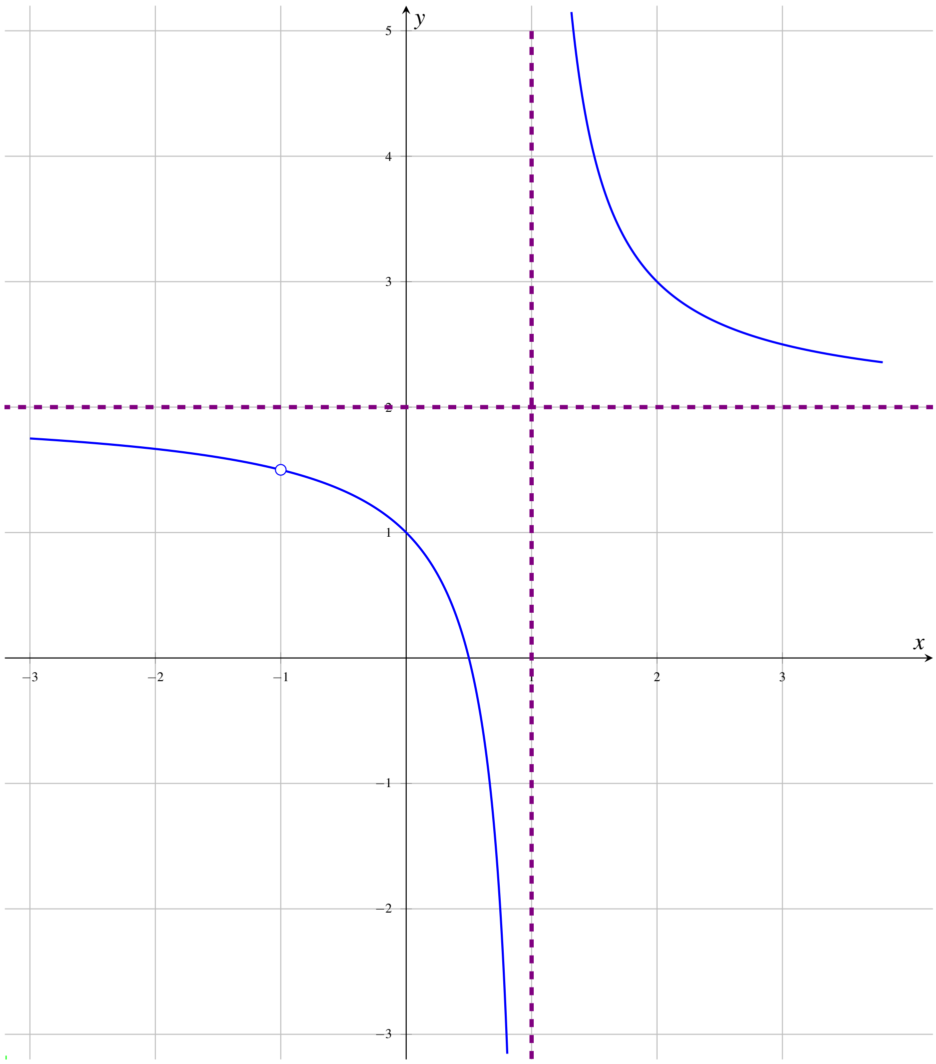
$$x = 2 \Rightarrow y = 1$$

$$A = \int_0^1 x^2 dx + \int_1^2 (2-x)dx = \frac{x^3}{3}\Big|_0^1 + (2x - \frac{x^2}{2})\Big|_1^2$$

$$= \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

3. [25 Points] Sketch the graph of the rational function $y = \frac{2x^2+x-1}{x^2-1}$. Please note that $y' = -\frac{1}{(x-1)^2}$ and $y'' = \frac{2}{(x-1)^3}$.

Solution:



Clearly y is undefined at $x=-1$. We can reaarange y as

$$y = \frac{2x-1}{x-1} = 2 + \frac{1}{x-1}$$

. Beside this, y' is undefined at$x = 1$ because $y' = -\frac{1}{(x-1)^2} = 0$. Therefore, the critical point is only $x = 1$. We can also see that y'' is not defined at $x = 1$.

Next we must find the asymptotes of the graph: $\lim_{x\to 1} y = \infty$. Hence $x = 1$ is an vertical asymptote of the graph. Moreover $\lim_{x\to\infty} y = 2$. So $y = 2$ is a horizontal asymptote of the graph. There is not any obligue asymptote.

We can also say that

(i) f is increasing on the intervals $(-\infty,-1)$ and $(1,\infty)$;

(ii) f is decreasing on the intervals $(-\infty,1)$ and $(1,\infty)$;

(iii) f is concave down on the interval $(-\infty,1)$;

(iv) f is concave up on the interval $(1,\infty)$; and

(v) There is not any local maximum or local minimum.

The table below summarises this informations:

x	$-\infty$	1	∞
$f'(x)$	-	-	
$f''(x)$	-	+	
$f(x)$