Math 113 Summer 2018 **Second Exam** July 24, 2018

Your Name / Ad - Soyad (**75 min.**) Signature / İmza

Student ID # / Öğrenci No (**use a blue pen!**)

Problem	1	2	3	4	Total
Points:	30	25	20	25	100
Score:					

Time limit is **75 minutes**. If you need more room on your exam paper, you may use the empty spaces on the paper. You have to show your. Answers without reasonable-even if your results are true- work will either get zero or very little credit.

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1. (a) (10 Points) Find the absolute maximum and absolute minimum values of $f(x) = \sqrt[3]{x}$ on the interval $-1 \le x \le 8$.

Solution: The function is continuous on the closed interval so its both absolute and minimum values exist. Hence

$$h(x) = \sqrt[3]{x} = x^{1/3} \Rightarrow h'(x) = \frac{1}{3}x^{-2/3}$$

$\Rightarrow$ a critical point at $x = 0$;

$h(-1) = -1, \quad h(0) = 0, \quad h(0) = 2;$

the absolute maximum is $\boxed{2}$ at $x = 8$

the absolute minimum is $\boxed{-1}$ at $x = -1$

p.72, pr.15

(b) (10 Points) Find a positive number x such that the sum of it and its multiplicative inverse $\frac{1}{x}$ is smallest possible.

Solution: We want the function $S(x) = x + \frac{1}{x}$ to be minimum for $x \in (0, \infty)$.

$$S'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}.$$

$$S'(x) = 0 \Rightarrow \frac{x^2-1}{x^2} = 0 \Rightarrow x^2 - 1 = 0 \quad x = \pm 1$$

Since $x = -1 \notin (0, \infty)$, we only consider $x = 1$.

$$S''(x) = \frac{2}{x^3} \Rightarrow S''(1) = \frac{2}{1^3} = 1 > 0$$

So local minimum value at $\boxed{x = 1}$.

Therefore, such positive number is $\boxed{x = 1}$.

p.94, pr.10

(c) (10 Points) For the function $f(x) = x^3 - x^2$ and interval $[-1, 2]$, find the value(s) of c satisfying $f'(c) = \frac{f(b) - f(a)}{b - a}$ in the conclusion of Mean Value Theorem.

Solution: When $f(x) = x^3 - x^2$ for $x \in [-1, 2]$, then $f'(x) = 3x^2 - 2x$, $f(-1) = -2$, $f(2) = 4$ and so

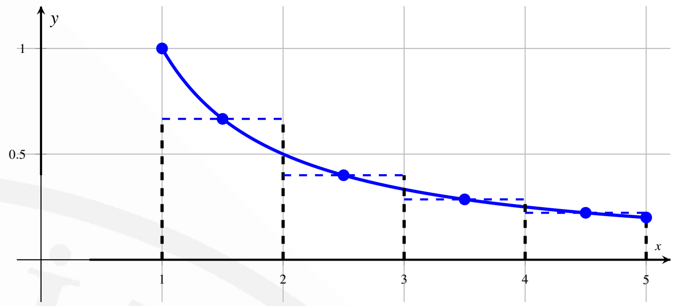
$$\frac{f(2) - f(-1)}{2 - (-1)} = f'(c)$$

$$2 = 3c^2 - 2c \Rightarrow c = \frac{1 \pm \sqrt{7}}{2}.$$

$\boxed{\frac{1+\sqrt{7}}{2}} \approx 1.22$ and $\boxed{\frac{1-\sqrt{7}}{2}} \approx -0.549$ are both in the interval $[-1, 2]$

p.94, pr.34

2. (a) (12 Points) Using four rectangles of equal width, approximate the area under $f(x) = \dfrac{1}{x}$ over the interval $[1,5]$ in which the heights of rectangles are determined by midpoint rule.

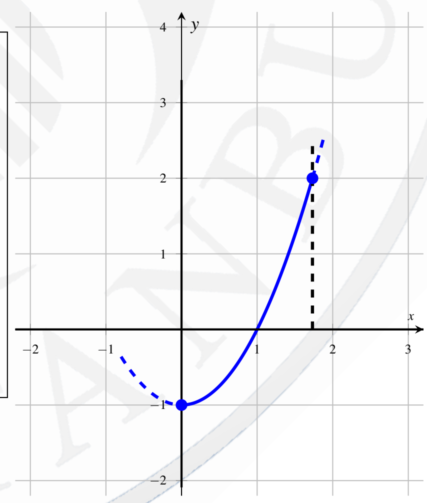


Solution: Here $a = 1$ and $b = 5$ and $n = 4$ and so $\Delta x = \dfrac{b-a}{n} = \dfrac{5-1}{4} = 1$ and $x_k = k\Delta x = 1 + k$. Therefore the partition is $P = \{x_0 = 1, x_1 = 2, x_2 = 3, x_3 = 4, x_4 = 5\}$. Therefore, an upper sum is

$$\begin{aligned}\sum_{k=0}^{3} \frac{1}{x_k}.(1) &= 1\left(f(\frac{3}{2}) + f(\frac{5}{2}) + f(\frac{7}{2}) + f(\frac{9}{2})\right)\\ &= \frac{2}{3} + \frac{2}{5} + \frac{2}{7} + \frac{2}{9} = \frac{1488}{3\cdot5\cdot7\cdot9} = \frac{496}{5\cdot7\cdot9}\\ &= \boxed{\frac{496}{315}}\end{aligned}$$

(Page 253, problem 3d)

(b) (13 Points) Over the interval $[0,\sqrt{3}]$, find the average value of $f(x) = x^2 - 1$.



Solution:

$$\begin{aligned}\text{av}(f) &= \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{\sqrt{3}-0}\int_0^{\sqrt{3}} (x^2-1)\,dx\\ &= \frac{1}{\sqrt{3}}\int_0^{\sqrt{3}} (x^2)\,dx - \frac{1}{\sqrt{3}}\int_0^{\sqrt{3}} (1)\,dx\\ &= \frac{1}{(\sqrt{3})}\frac{(\sqrt{3})^3}{3} - \frac{1}{\sqrt{3}}(\sqrt{3}-0) = 1 - 1 = \boxed{0}\end{aligned}$$

p.72, pr.55

3. (a) (10 Points) Find the integral $\displaystyle\int \frac{8x^3}{(x^4+1)^2}\,dx$.

Solution: Let $u = x^4+1$. Then $du = 4x^3$ and so

$$\int \frac{8x^3}{(x^4+1)^2}\,dx = 2\int \underbrace{\frac{1}{(x^4+1)^2}}_{1/u^2}\underbrace{4x^3\,dx}_{du} = 2\int \frac{1}{u^2}\,du$$

$$= 2\frac{u^{-2+1}}{-2+1} + C = 2\frac{u^{-1}}{-1} + C$$

$$= \boxed{-\frac{2}{x^4+1} + C}$$

p.83, pr.52

(b) (10 Points) Let $F(x) = \displaystyle\int_{\sqrt{x}}^{1} \cos(t^2)\,dt$.

$F(1) =$

$F'(x) =$

$F'(\pi) =$

Solution: First, $F(1) = \displaystyle\int_{\sqrt{1}}^{1}\cos(t^2)\,dt = \int_1^1 \cos(t^2)\,dt = \boxed{0}$. Next, by the Fundamental Theorem of Calculus, Part I, we have

$$F'(x) = \frac{d}{dx}\int_{\sqrt{x}}^{1}\cos(t^2)\,dt = \frac{d}{dx}\left(-\int_1^{\sqrt{x}}\cos(t^2)\,dt\right) = -\frac{d}{dx}\left(\int_1^{\sqrt{x}}\cos(t^2)\,dt\right)$$

$$= -\cos((\sqrt{x})^2)\frac{d}{dx}(\sqrt{x})$$

$$= -(\cos x)\frac{1}{2\sqrt{x}} = \boxed{-\frac{\cos x}{2\sqrt{x}}}$$

Finally, $F'(\pi) = -\dfrac{\cos\pi}{2\sqrt{\pi}} = \boxed{-\dfrac{1}{2\sqrt{\pi}}}$

p.83, pr.52

4. Suppose $y = \dfrac{x^2}{x+1} = x - 1 + \dfrac{1}{x+1}$. You may assume that $y' = \dfrac{x(x+2)}{(x+1)^2}$ and $y'' = \dfrac{2}{(x+1)^3}$.

   (a) (7 Points) Give the *asymptotes*.

Solution: The graph has two asymptotes. One is vertical, the other is oblique. The line $x = -1$ is a vertical asymptote as $\displaystyle\lim_{x\to -1^{\pm}} \frac{x^2}{x+1} = \pm\infty$. Also since $\dfrac{x^2}{x+1} = x - 1 + \dfrac{1}{x+1}$ and $\displaystyle\lim_{x\to\pm\infty} \frac{1}{x+1} = 0$, the line $y = x - 1$ is an oblique asymptote.

p.212, pr.85

   (b) (5 Points) Find the *intervals* where the graph is *increasing* and *decreasing*. Find the *local maximum* and *minimum* values.

Solution: We have $y' = 1 - \dfrac{1}{(x+1)^2} = 0$ if and only if $(x+1)^2 = 1$, that is iff $x = 0$ and $x = -2$ are the critical points. Note that

$$y' \text{ is } \begin{cases} > 0, & \text{on } (-\infty, -2) \cup (0, \infty) \qquad \text{y is incresing} \\[2ex] < 0, & \text{on } (-2, -1) \cup (-1, 0) \qquad \text{y is decresing} \end{cases}$$

Thus, y is decreasing on $(-2, -1) \cup (-1, 0)$ and increasing on $(-\infty, -2) \cup (0, \infty)$. Moreover, the point $(-2, -4)$ is a point of local maximum and $(0, 0)$ is a point of local minimum. p.212, pr.85

(c) (5 Points) Determine where the graph is *concave up* and *concave down*, and find any *inflection points*.

(d) (8 Points) *Sketch the graph* of the function. Label the asymptotes, critical points and the inflection points.

$$y = f(x) = \frac{x^2}{x+1}$$