

Solutions to Exercise Sheet 1 are now on my website.

In the exams, you will typically not be told if an equation is linear, separable, exact, homogeneous, etc – you should be able to determine this yourself. You can use Exercises 15 and 16 to practise.

Exercise 15 (First Order ODEs). Find the general solutions of the following ODEs:

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| (a) $9yy' + 4x = 0$. | (j) $e^{\frac{x}{y}}(y - x)\frac{dy}{dx} + y(1 + e^{\frac{x}{y}}) = 0$. |
| (b) $y' + (x + 1)y^3 = 0$. | (k) $(2x + 3y)dx + (3x + 2y)dy = 0$. |
| (c) $\frac{dx}{dt} = 3t(x + 1)$. | (l) $(x^3 + \frac{y}{x})dx + (y^2 + \ln x)dy = 0$. |
| (d) $y' + \csc y = 0$. | (m) $(e^x \sin y + \tan y)dx + (e^x \cos y + x \sec^2 y)dy = 0$. |
| (e) $x' \sin 2t = x \cos 2t$. | (n) $ydx + (2x - ye^y)dy = 0$. |
| (f) $y' = (y - 1) \cot x$. | (o) $xy' + y = y^{-2}$. |
| (g) $\frac{dy}{dx} + (\frac{2x+1}{x})y = e^{-2x}$. | (p) $y' = y(xy^3 - 1)$. |
| (h) $(3x^2 + y^2)dx - 2xydy = 0$. | (q) $(1 + x^2)y' = 2xy(y^3 - 1)$. |
| (i) $y' = \frac{y}{x} + \tan(\frac{y}{x})$. | |

Exercise 16 (Initial Value Problems). Solve the following IVPs:

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| (a) $\begin{cases} y' = x^3 e^{-y} \\ y(2) = 0 \end{cases}$ | (e) $\begin{cases} \frac{dy}{dx} = \frac{10}{(x+y)e^{x+y}} - 1 \\ y(0) = 0 \end{cases}$ | (i) $\begin{cases} (xy + 1)ydx + (2y -)dy = 0 \\ y(0) = 3 \end{cases}$ |
| (b) $\begin{cases} y\frac{dy}{dx} = 4x(y^2 + 1)^{\frac{1}{2}} \\ y(0) = 1 \end{cases}$ | (f) $\begin{cases} (4x^2 - 2y^2)y' = 2xy \\ y(3) = -5 \end{cases}$ | (j) $\begin{cases} y' - \frac{1}{x}y = y^2 \\ y(1) = 2 \end{cases}$ |
| (c) $\begin{cases} y' = y \cot x \\ y(\frac{\pi}{2}) = 2 \end{cases}$ | (g) $\begin{cases} (x - y)dx + (3x + y)dy = 0 \\ y(3) = -2 \end{cases}$ | |
| (d) $\begin{cases} y' + 3(y - 1) = 2x \\ y(0) = 1 \end{cases}$ | (h) $\begin{cases} \frac{dy}{dx} = \frac{x^3 - xy^2}{x^2y} \\ y(1) = 1 \end{cases}$ | |

Exercise 17 (Homogeneous Second Order Linear ODEs with constant coefficients). Solve the following IVPs:

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| (a) $\begin{cases} y'' - 3y' + 2y = 0 \\ y(0) = 1 \\ y'(0) = 1 \end{cases}$ | (b) $\begin{cases} y'' + 4y' + 3y = 0 \\ y(0) = 2 \\ y'(0) = -1 \end{cases}$ | (c) $\begin{cases} y'' + 3y' = 0 \\ y(0) = -2 \\ y'(0) = 3 \end{cases}$ | (d) $\begin{cases} y'' + 5y' + 3y = 0 \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$ |
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Exercise 18 (Fundamental Sets of Solutions). In each of the following: Verify that y_1 and y_2 are solutions of the given ODE; calculate the Wronskian of y_1 and y_2 ; and determine if they form a fundamental set of solutions.

- (a) $t^2y'' - 2y = 0$; $y_1(t) = t^2$, $y_2(t) = t^{-1}$
- (b) $y'' + 4y = 0$; $y_1(t) = \cos 2t$, $y_2(t) = \sin 2t$
- (c) $y'' - 2y + y = 0$; $y_1(t) = e^t$, $y_2(t) = te^t$
- (d) $(1 - x \cot x)y'' - xy' + y = 0$ ($0 < x < \pi$); $y_1(x) = x$, $y_2(x) = \sin x$