İSTANBUL OKAN ÜNİVERSİTESİ
MÜHENDİSLİK FAKÜLTESİ
MÜHENDİSLİK TEMEL BİLİMLERİ BÖLÜMÜ

2018–19 **MATH216 Mathematics IV – Exercise Sheet 7** N. Course

Hint for 27(j)-(k): Note that since $\mathcal{L}[tf(t)] = (-1)\frac{dF}{ds}$, we have that $-\mathcal{L}^{-1}\left[\frac{dF}{ds}\right] = tf(t)$ and thus $f(t) = -\frac{1}{t}\mathcal{L}^{-1}\left[\frac{dF}{ds}\right]$.

Exercise 28 (The Laplace Transform). Use the definition $\mathcal{L}[f](s) = \int_0^\infty e^{-st} f(t)\,dt$ to prove that the following identities are true. The first one is done for you.

(ω) $\mathcal{L}[1](s) = \frac{1}{s}$

$$\mathcal{L}[1](s) = \int_0^\infty e^{-st}(1)\,dt = \lim_{A\to\infty}\int_0^A e^{-st}(1)\,dt = \lim_{A\to\infty}\left[-\frac{e^{-st}}{s}\right]_0^A = \lim_{A\to\infty}\left(-\frac{e^{-sA}}{s} + \frac{e^0}{s}\right) = \frac{1}{s}.$$

(a) $\mathcal{L}[t^2](s) = \frac{2}{s^3}$ for $s > 0$

(b) $\mathcal{L}[\cos at](s) = \frac{s}{s^2+a^2}$ for $s > 0$

(c) $\mathcal{L}[\sinh at](s) = \frac{a}{s^2-a^2}$ for $s > a$

(d) $\mathcal{L}[\cosh at](s) = \frac{s}{s^2-a^2}$ for $s > a$

(e) $\mathcal{L}[f(ct)](s) = \frac{1}{c}\mathcal{L}[f]\left(\frac{s}{c}\right)$

(f) $\frac{d}{ds}\mathcal{L}[f](s) = -\mathcal{L}[tf(t)](s)$

Exercise 29 (The Laplace Transform). Use the Laplace Transform to solve the following initial value problems:

(a) $\begin{cases} x'' + 4x = 0 \\ x(0) = 5 \\ x'(0) = 0 \end{cases}$

(b) $\begin{cases} x'' - x' - 2x = 0 \\ x(0) = 0 \\ x'(0) = 2 \end{cases}$

(c) $\begin{cases} x'' + 9x = 1 \\ x(0) = 0 \\ x'(0) = 0 \end{cases}$

(d) $\begin{cases} x'' + 6x' + 25x = 0 \\ x(0) = 2 \\ x'(0) = 3 \end{cases}$

(e) $\begin{cases} x'' - 6x' + 8x = 2 \\ x(0) = 0 \\ x'(0) = 0 \end{cases}$

(f) $\begin{cases} x'' - 4x = 3t \\ x(0) = 0 \\ x'(0) = 0 \end{cases}$

(g) $\begin{cases} x'' + 4x' + 8x = e^{-t} \\ x(0) = 0 \\ x'(0) = 0 \end{cases}$

(h) $\begin{cases} x^{(4)} + 8x'' + 16x = 0 \\ x(0) = x'(0) = x''(0) = 0 \\ x^{(3)}(0) = 1 \end{cases}$

(i) $\begin{cases} x^{(4)} + 2x'' + x = e^{2t} \\ x(0) = x'(0) = x''(0) = x^{(3)}(0) = 1 \end{cases}$

(j) $\begin{cases} x^{(3)} + 4x'' + 5x' + 2x = 10\cos t \\ x(0) = x'(0) = 0 \\ x''(0) = 3 \end{cases}$

(k) $\begin{cases} x'' + 4x' + 13x = te^{-t} \\ x(0) = 0 \\ x'(0) = 2 \end{cases}$

(l) $\begin{cases} x'' + x = \sin 2t \\ x(\frac{\pi}{2}) = 2 \\ x'(\frac{\pi}{2}) = 0 \end{cases}$

$f(t)$	$F(s) = \mathcal{L}[f](s)$	
1	$\frac{1}{s}$	$s > 0$
e^{at}	$\frac{1}{s-a}$	$s > a$
$t^n \quad (n \in \mathbb{N})$	$\frac{n!}{s^{n+1}}$	$s > 0$
$\sin at$	$\frac{a}{s^2+a^2}$	$s > 0$
$\cos at$	$\frac{s}{s^2+a^2}$	$s > 0$
$\sinh at$	$\frac{a}{s^2-a^2}$	$s > \lvert a\rvert$
$\cosh at$	$\frac{s}{s^2-a^2}$	$s > \lvert a\rvert$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$	$s > a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$	$s > a$
$t^n e^{at} \quad (n \in \mathbb{N})$	$\frac{n!}{(s-a)^{n+1}}$	$s > a$
$u_c(t)$	$\frac{e^{-cs}}{s}$	$s > 0$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$	
$e^{ct}f(t)$	$F(s-c)$	
$f(ct) \quad (c > 0)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$	
$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$	
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	

$\mathcal{L}[f'](s) = s\mathcal{L}[f](s) - f(0)$

$\mathcal{L}[f''](s) = s^2\mathcal{L}[f](s) - sf(0) - f'(0)$

$\mathcal{L}[f^{(n)}](s) = s^n\mathcal{L}[f](s) - s^{n-1}f(0) - \ldots - sf^{(n-2)}(0) - f^{(n-1)}(0)$