2018–19 **MATH216 Mathematics IV – Exercise Sheet 8** N. Course

Exercise 30 (Systems of Linear Equations). Find the general solutions to the following systems of ODEs:

(a) $\mathbf{x}' = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \mathbf{x}$

(b) $\mathbf{x}' = \begin{bmatrix} -3 & 2 \\ -3 & 4 \end{bmatrix} \mathbf{x}$

(c) $\mathbf{x}' = \begin{bmatrix} 3 & -1 \\ 5 & -3 \end{bmatrix} \mathbf{x}.$

(d) $\begin{cases} x' = 4x - y \\ y' = x + 2y \end{cases}$

(e) $\begin{cases} x' = 3x - y \\ y' = 4x - y \end{cases}$

(f) $\begin{cases} x' = 5x + 4y \\ y' = -x + y \end{cases}$

(g) $\begin{cases} x' = 3x + 2y \\ y' = -5x + y \end{cases}$

(h) $\begin{cases} x' = x - 4y \\ y' = x + y \end{cases}$

(i) $\begin{cases} x' = x - 3y \\ y' = 3x + y \end{cases}$

(j) $\begin{cases} x' = 4x - 2y \\ y' = 5x + 2y \end{cases}$

(k) $\begin{cases} x' = x + y - z \\ y' = 2x + 3y - 4z \\ z' = 4x + y - 4z \end{cases}$

(l) $\begin{cases} x' = x - y - z \\ y' = x + 3y + z \\ z' = -3x + y - z \end{cases}$

(m) $\begin{cases} x' = 3x + y + z \\ y' = 3y + z \\ z' = 6z \end{cases}$

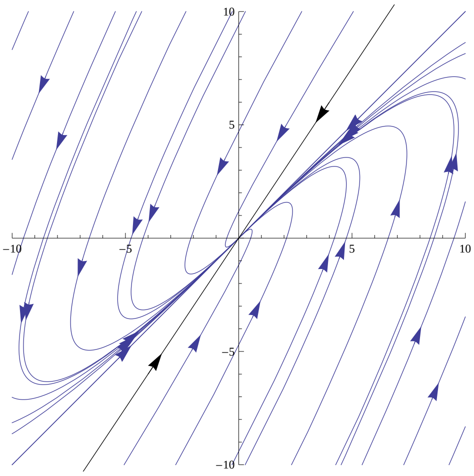
(n) $\begin{cases} x' = 2x + y - z \\ y' = -4x - 3y - z \\ z' = 4x + 4y + 2z \end{cases}$

Exercise 31 (Initial Value Problems). Solve the following IVPs:

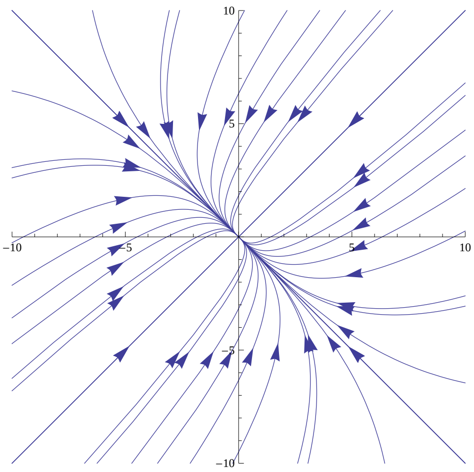
(a) $\begin{cases} \mathbf{x}' = \begin{bmatrix} 3 & 4 \\ 3 & 2 \end{bmatrix} \mathbf{x} \\ \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{cases}$

(b) $\begin{cases} \mathbf{x}' = \begin{bmatrix} 4 & -3 \\ 6 & -7 \end{bmatrix} \mathbf{x} \\ \mathbf{x}(0) = \begin{bmatrix} 8 \\ 0 \end{bmatrix} \end{cases}$

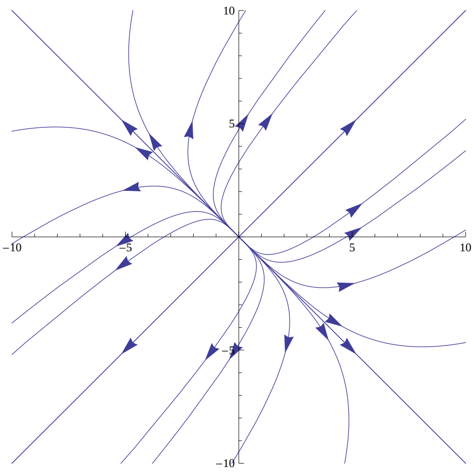
(c) $\begin{cases} x' = 3x + z \\ y' = 9x - y + 2z \\ z' = -9x + 4y - z \\ x(0) = 0 \\ y(0) = 0 \\ z(0) = 17 \end{cases}$



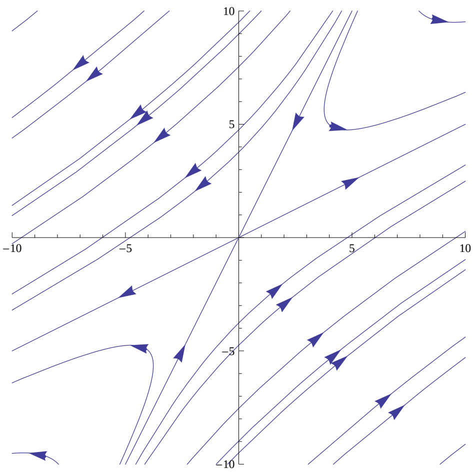
(i) Stable node



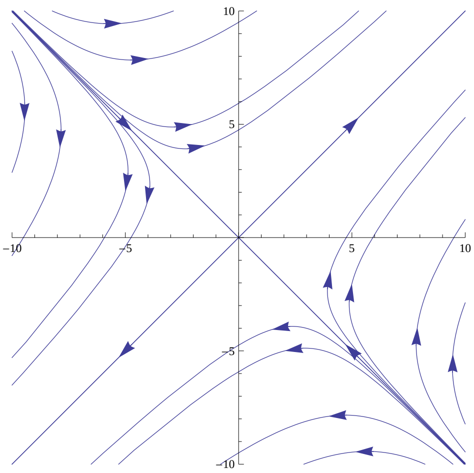
(ii) Stable node



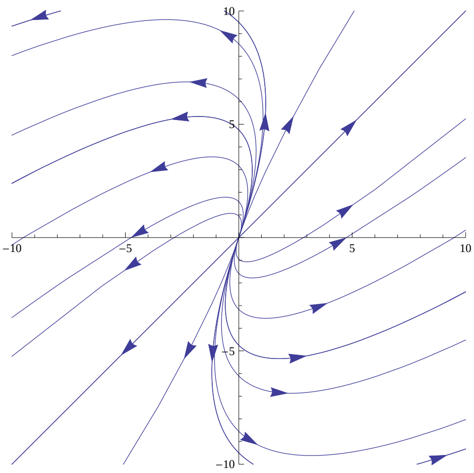
(iii) Unstable node



(iv) Saddle



(v) Saddle



(vi) Unstable node

Exercise 32 (Phase Plots). Match the system of ODEs $\mathbf{x}' = A\mathbf{x}$ with the correct phase plot shown above, for each matrix A below. You are given eigenvalues and eigenvectors for each matrix. The first one is done for you.

(ω) $A = \begin{bmatrix} 3 & 5 \\ 5 & 3 \end{bmatrix}$; $r_1 = 8$, $r_2 = -2$; $\xi^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\xi^{(2)} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

> Since one eigenvalue is positive and one is negative, $\mathbf{x}' = A\mathbf{x}$ must have a saddle point. So the phase plot must be either (iv) or (v).
>
> The phase plot must also have straight lines in the directions of the eigenvectors. So it must be (v).

(a) $A = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix}$; $r_1 = -2$, $r_2 = -1$; $\xi^{(1)} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $\xi^{(2)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(b) $A = \begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}$; $r_1 = -3$, $r_2 = -1$; $\xi^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\xi^{(2)} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

(c) $A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$; $r_1 = 4$, $r_2 = 2$; $\xi^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\xi^{(2)} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

(d) $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$; $r_1 = 8$, $r_2 = 2$; $\xi^{(1)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\xi^{(2)} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

(e) $A = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}$; $r_1 = 2$, $r_2 = -1$; $\xi^{(1)} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\xi^{(2)} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.