MATH 216 MATHEMATICS IV
Homework 2

A. Find the general solutions of the following homogeneous differential equations:

1. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0.$
2. $4\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 5y = 0.$
3. $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} + \frac{dy}{dx} + 6y = 0.$
4. $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 0.$
5. $\frac{d^3y}{dx^3} - 4\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 18y = 0.$
6. $\frac{d^2y}{dx^2} + 16y = 0.$
7. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y = 0.$
8. $\frac{d^7y}{dx^7} + 2\frac{d^5y}{dx^5} + \frac{d^3y}{dx^3} = 0.$

B. Find the linear, constant coefficient and homogeneous differential equations whose general solutions are given below.

1. $x_H(t) = c_1 + c_2t + c_3e^{3t}\sin t + c_4e^{3t}\cos t + c_5e^{3t}\sin 2t + c_6e^{3t}\cos 2t.$
2. $x_H(t) = c_1e^t + c_2te^t + c_3e^{2t}\sin t + c_4e^{2t}\cos t + c_5e^{2t}t\sin 2t + c_6e^{2t}t\cos 2t.$
3. $x_H(t) = c_1e^{2t} + c_2te^{2t} + c_3t^2e^{2t} + c_4e^{-t}\sin 3t + c_5e^{-t}\cos 3t.$

C. Solve the following problems:

1. Given that $\sin t$ is a solution of
$$\frac{d^4y}{dt^4} + 2\frac{d^3y}{dt^3} + 6\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = 0.$$
Find the general solution.
2. Write y_p by using the method of undetermined coefficients for
$$\frac{d^2y}{dx^2} + y = 3x^2 - 4\sin x.$$
3. Write x_p by using the method of undetermined coefficients for
$$\frac{d^4x}{dt^4} - 16x = t^2\sin 2t + t^4e^{2t}.$$

D. Find the general solutions for the following differential equations.

1. $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + y = 3xe^x.$
2. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 3\sin 2x.$
3. $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 3x = 1 + te^t.$
4. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x.$

5. $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} - 3x = -3te^{-t}$.

6. $\frac{d^2x}{dt^2} + 9x = 2\cos 3t + 2\sin 3t$.

7. $\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} = 3x^2 + 4\sin x - 2\cos x$.

8. $\frac{d^2x}{dt^2} + 9x = 9\sec^2 3t, \quad 0 < t < \frac{\pi}{6}$.
   A: $x(t) = c_1\cos 3t + c_2\sin 3t + \sin 3t \ln|\sec 3t + \tan 3t| - 1$.

9. $\frac{d^2x}{dt^2} + x = \sec t$.
   A: $x(t) = c_1\cos t + c_2\sin t + \cos t \ln|\cos t| + t\sin t$.

10. $\frac{d^2x}{dt^2} + 4x = 3\csc^2 2t,\ 0 < t < \frac{\pi}{2}$.
    A: $x(t) = c_1\cos 2t + c_2\sin 2t - \frac{3}{2}t\cos 2t + \frac{3}{4}\sin 2t \ln|\sin 2t|$.

11. $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \frac{1}{1+e^x}$.
    A: $y(x) = c_1e^{-x} + c_2e^{-2x} + \left(e^{-x} + e^{-2x}\right)\ln|1 + e^x|$.

12. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \ln x; \ \ x > 0$.
    A: $y(x) = c_1e^x + c_2xe^x + \frac{1}{6}(x^3e^x \ln x - \frac{5}{6}x^3e^x)$.

E. Solve the following initial value problems:

1. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0; y(0) = 3, y'(0) = 5$.

2. $9\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + y = 0; y(0) = 3, y'(0) = 0$.

3. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 29y = 0;\ y(0) = 0, y'(0) = 5$.

4. $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 8y = 0; y(0) = 2, y'(0) = 0, y''(0) = 0$.

5. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 9x^2 + 4; y(0) = 7, y'(0) = -3$.

6. $\frac{d^2x}{dt^2} + 4x = t^2 + 3e^t; x(0) = 0, x'(0) = 2$.
   A: $x(t) = -\frac{19}{40}\cos 2t + \frac{7}{10}\sin 2t + \frac{1}{4}t^2 - \frac{1}{8} + \frac{3}{5}e^t$.

7. $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = te^t + 4; x(0) = 1, x'(0) = 1$.
   A: $x(t) = -3e^t + 4te^t + \frac{1}{6}t^3e^t + 4$.

8. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 4e^{-x}\cos 2x; y(0) = 1, y'(0) = 0$.
   A: $y(x) = e^{-x}\cos 2x + \frac{1}{2}e^{-x}\sin 2x + xe^{-x}\sin 2x$.

9. $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x^{-2}e^{-2x}, 0 < x; y(1) = \frac{2}{e^2}, y'(1) = -\frac{4}{e^2}$.
   A: $y(x) = e^{-2x} + xe^{-2x} - e^{-2x}\ln x$.

10. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{1+x^2}; y(0) = 3, y'(0) = 1$.
    A: $y(x) = 3e^x - 2xe^x - \frac{1}{2}e^x \ln\left(1 + x^2\right) + xe^x \arctan x$.