Your Name / Adınız - Soyadınız

Your Signature / İmza

Student ID # / Öğrenci No

Professor's Name / Öğretim Üyesi

Your Department / Bölüm

- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct. **Show your work in evaluating any limits, derivatives**.
- Place $\boxed{\text{a box around your answer}}$ to each question.
- Use a **BLUE ball-point pen** to fill the cover sheet. Please make sure that your exam is complete.
- Time limit is 70 min.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	30	
3	25	
4	25	
Total:	100	

1. $\boxed{\text{20 Points}}$ Find the inverse Laplace transform of $F(s) = \dfrac{3s}{s^2 - s - 6}$.

Solution: We have using partial fractions

$$\frac{3s}{s^2-s-6} = \frac{3s}{(s-3)(s+2)} = \frac{a}{s-3} + \frac{b}{s+2} = \frac{a(s+2)+b(s-3)}{(s-3)(s+2)} = \frac{(a+b)s+(2a-3b)}{(s-3)(s+2)}$$

Equating like powers in the numerators we find $a+b=3$ and $2a-3b=0$. Thus $a=9/5$ and $b=6/5$. Thus

$$\mathscr{L}^{-1}\{F(s)\} = \frac{9}{5}\mathscr{L}^{-1}\left\{\frac{1}{s-3}\right\} + \frac{6}{5}\mathscr{L}^{-1}\left\{\frac{1}{s+2}\right\} = \frac{9}{5}e^{3t} + \frac{6}{5}e^{-2t}$$

p.324, pr.4

2. $\boxed{\text{30 Points}}$ Solve the initial value problem

$$y'' + y = f(t), \quad y(0) = 0, \quad y'(0) = 0$$

where f is given by

$$f(t) = \begin{cases} t & 0 \le t < 3 \\ 0 & t \ge 3 \end{cases}$$

Solution: The function f can be written as $f(t) = t - (t-3)u_3(t) - 3u_3(t)$. Taking the Laplace transform of both sides, we have

$$\mathscr{L}\{y'' + y\} = \mathscr{L}\{t - (t-3)u_3(t) - 3u_3(t)\} \Rightarrow \mathscr{L}\{y''\} + \mathscr{L}\{y\} = \mathscr{L}\{t\} - \mathscr{L}\{(t-3)u_3(t)\} - 3\mathscr{L}\{u_3(t)\}$$

$$s^2\mathscr{L}\{y\} - sy(0) - y'(0) + \mathscr{L}\{y\} = \frac{1}{s^2} - \frac{e^{-3s}}{s^2} - \frac{3e^{-s}}{s}$$

$$Y(s)(s^2+1) = \frac{1}{s^2} - \frac{e^{-3s}}{s^2} - \frac{3e^{-3s}}{s} \Rightarrow Y(s) = \frac{1}{s^2(s^2+1)} - \frac{e^{-3s}}{s^2(s^2+1)} - \frac{3e^{-3s}}{s(s^2+1)}$$

$$Y(s) = \frac{1}{s^2} - \frac{1}{s^2+1} - \left(\frac{1}{s^2} - \frac{1}{s^2+1}\right)e^{-3s} - \left(\frac{1}{s} - \frac{s}{s^2+1}\right)e^{-3s}$$

$$y(t) = \mathscr{L}^{-1}\{Y(s)\} = \mathscr{L}^{-1}\left\{\frac{1}{s^2}\right\} - \mathscr{L}^{-1}\left\{\frac{1}{s^2+1}\right\} - \mathscr{L}^{-1}\left\{\left(\frac{1}{s^2} - \frac{1}{s^2+1}\right)e^{-3s}\right\} - \mathscr{L}^{-1}\left\{\left(\frac{1}{s} - \frac{s}{s^2+1}\right)e^{-3s}\right\}$$

$$\boxed{y(t) = t - \sin(t) - ((t-3) - \sin((t-3)))\,u_3(t) - (1 - \cos(t-3))\,u_3(t)}$$

p.491, pr.86

3. $\boxed{\text{25 Points}}$ Find the inverse Laplace transform of

$$F(s) = \frac{2e^{-4s}}{s^2-4}.$$

Solution: First use partial fractions

$$\frac{2}{s^2-4} = \frac{A}{s-2} + \frac{B}{s+2} = \frac{A(s+2)+B(s-2)}{(s-2)(s+2)} = \frac{(A+B)s+2A-2B}{(s-2)(s+2)} \Rightarrow \begin{cases} A+B=0 \\ 2A-2B=2 \end{cases} \Rightarrow \begin{cases} A=\frac{1}{2} \\ B=-\frac{1}{2} \end{cases}$$

Hence we have

$$\frac{2}{s^2-4} = \frac{1/2}{s-2} + \frac{-1/2}{s+2} \Rightarrow F(s) = \frac{2e^{-4s}}{s^2-4} = \frac{1/2}{s-2}e^{-4s} + \frac{-1/2}{s+2}e^{-4s} = \frac{1}{2}\left(\frac{e^{-4s}}{s-2} - \frac{e^{-4s}}{s-2}\right)$$

Therefore

$$\mathscr{L}^{-1}\{F(s)\} = \frac{1}{2}\mathscr{L}^{-1}\left\{\frac{e^{-4s}}{s-2}\right\} - \frac{1}{2}\mathscr{L}^{-1}\left\{\frac{e^{-4s}}{s+2}\right\} = \frac{1}{2}e^{2(t-4)}u_4(t) - \frac{1}{2}e^{-2(t-4)}u_4(t)$$

p.333, pr.22

4. $\boxed{\text{25 Points}}$ Find the Laplace transform of

$$f(t) = \int_0^t (t-\tau)e^{5\tau}\,\mathrm{d}\tau.$$

Solution: With $f(t) = t$ and $g(t) = e^{5t}$ convolution theorem states that the Laplace transform of the convolution of f and g is the product of their Laplace transforms:

$$\mathscr{L}\{f(t)\} = \mathscr{L}\left\{\int_0^t (t-\tau)e^{5\tau}\,\mathrm{d}\tau\right\} = \mathscr{L}\{f*g\} = \mathscr{L}\{f\}\mathscr{L}\{g\} = \mathscr{L}\{t\}\mathscr{L}\left\{e^{5t}\right\} = \frac{1}{s^2}\frac{1}{s-5}.$$

p.355, pr.6

$f(t)$	$F(s) = \mathscr{L}[f](s)$	
1	$\frac{1}{s}$	$s>0$
e^{at}	$\frac{1}{s-a}$	$s>a$
$t^n \quad (n\in\mathbb{N})$	$\frac{n!}{s^{n+1}}$	$s>0$
$\sin at$	$\frac{a}{s^2+a^2}$	$s>0$
$\cos at$	$\frac{s}{s^2+a^2}$	$s>0$
$\sinh at$	$\frac{a}{s^2-a^2}$	$s>\lvert a\rvert$
$\cosh at$	$\frac{s}{s^2-a^2}$	$s>\lvert a\rvert$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$	$s>a$
$t^n e^{at} \quad (n\in\mathbb{N})$	$\frac{n!}{(s-a)^{n+1}}$	$s>a$
$u_c(t)$	$\frac{e^{-cs}}{s}$	$s>0$
$u_c(t)f(t-c)$	$e^{-cs}F(s)$	
$e^{ct}f(t)$	$F(s-c)$	
$\delta(t-c)$	e^{-cs}	
$f(ct) \quad (c>0)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$	
$\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$	
$t^n f(t)$	$(-1)^n F^{(n)}(s)$	