ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science

MATH 325
Introduction to Abstract Algebra I

Final

Jan 8, 2007

14:00-16:00

Surname : ____________________

Name : ____________________

ID # : ____________________

Department : ____________________

Section : ____________________

Instructor : ____________________

Signature : ____________________

- The exam consists of 6 questions.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might not get full credit.
- Calculators are not allowed.

GOOD LUCK!

Please do not write below this line.

Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
20	20	20	20	20	20	120

1. (20 pts.) Mark each of the following assertions True (**T**) or False (**F**). Justify your answer: give a proof or a counterexample.

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a) In the group $U(30)$, the subgroup $H = \{1, 11\}$ has 4 cosets.

b) If $\mu = (1245)(36)$ then $|S_6 : \langle \mu \rangle| = 180$.

c) $\mathbb{Z}_3 \oplus \mathbb{Z}_5$ is isomorphic to $\mathbb{Z}_{15}$.

d) $\mathbb{Z}_3 \oplus \mathbb{Z}_3 \oplus \mathbb{Z}_3$ has an element of order 9.

e) Let $G = U(32)$ and $K = \{1, 15\}$. Then $G/K \cong \mathbb{Z}_8$.

2. Let $G = \{(1), (12)(34), (1234)(56), (13)(24), (1432)(56), (56)(13), (14)(23), (24)(56)\}$.
a) (6 pts.) Find the stabilizer of 1 and the orbit of 1.
b) (7 pts.) Find the stabilizer of 3 and the orbit of 3.
c) (7 pts.) Find the stabilizer of 5 and the orbit of 5.

3.

a) (10 pts.) Determine the number of elements of order 10 in $\mathbb{Z}_{100} \oplus \mathbb{Z}_{25}$.

b) (10 pts.) Determine the number of cyclic subgroups of order 10 in $\mathbb{Z}_{100} \oplus \mathbb{Z}_{25}$.

4.

a) (10 pts.) Suppose that H and K are subgroups of a group G. If $|H| = 12$ and $|K| = 35$, find $|H \cap K|$.

b) (10 pts.) Let G be a group of order 25. Prove that G is cyclic or $g^5 = e$ for all g in G.

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5. Let $G = \mathbb{Z}_4 \oplus \mathbb{Z}_4$, $H = \{(0,0), (2,0), (0,2), (2,2)\}$, and $K = \langle (1,2) \rangle$.
a) (10 pts.) Is G/H isomorphic to $\mathbb{Z}_4$ or $\mathbb{Z}_2 \oplus \mathbb{Z}_2$?
b) (10 pts.) Is G/K isomorphic to $\mathbb{Z}_4$ or $\mathbb{Z}_2 \oplus \mathbb{Z}_2$?

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6. (15 pts.)
a) (10 pts.) Prove that $G_1 \oplus G_2$ is isomorphic to $G_2 \oplus G_1$.(Hint: Consider the map $(a, b) \mapsto (b, a)$)
b) (10 pts.) If $G_1 \oplus G_2$ is cyclic, prove that G_1 and G_2 are cyclic.
