# ÇANKAYA UNIVERSITY
Department of Mathematics and Computer Science

**MATH 325**
**Introduction to Abstract Algebra I**
$2^{nd}$ Midterm
Dec 4, 2006
17:40-19:10

Surname : ____________________
Name : ____________________
ID # : ____________________
Department : ____________________
Section : ____________________
Instructor : ____________________
Signature : ____________________

- The exam consists of 6 questions. You have to SOLVE the first problem and eliminate only one problem from the others.
- Please read the questions carefully and write your answers under the corresponding questions. Be neat.
- Show all your work. Correct answers without sufficient explanation might not get full credit.
- Calculators are not allowed.

*GOOD LUCK!*

Please do not write below this line.

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | TOTAL |
|---|---|---|---|---|---|---|
| | | | | | | |
| 25 | 20 | 20 | 20 | 20 | 20 | 105 |

**1.** (20 pts.) Mark each of the following assertions True (**T**) or False (**F**). Justify your answer: give a proof or a counterexample.

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a) $S_4$ is abelian.

b) The set of $2$-cycles in $S_4$ is not a subgroup of $S_4$.

c) There is no permutation $\beta$ such that $\beta^2 = (13579)\,(268)$ in $S_9$.

d) In $S_4$, there are exactly six elements of order $3$.

e) There is an element of order $10$ in $A_9$.

**2.** Find the centralizer of $(213)$
a) in $S_3$,
b) in $A_4$.

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**3.**

a) Find the order of

$$\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 2 & 5 & 4 & 9 & 8 & 6 & 7 & 13 & 3 & 12 & 11 & 10 & 1 \end{pmatrix}$$

and hence express $\tau^{245}$ in cycle notation.

b) In $S_{10}$, let $\beta = (13)\,(17)\,(265)\,(289)$. Find an element in $S_{10}$ that commutes with $\beta$ but is not a power of $\beta$. .

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**4.** Consider the permutation

$$\sigma = (1235)(2467)$$

a) (4 pts.) Find $\sigma(4)$ and $\sigma^2(4)$.
b) (4 pts.) Find $\sigma^{-1}$.
c) (4 pts.) Write $\sigma$ as a product of dijoint cycles (double check your answer with part a)).
d) (4 pts.) Find the order of $\sigma$
e) (4 pts.) Decide if $\sigma$ can be written as a product of a $3$-cycle and a $4$-cycle.

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**5.** Prove that the map $\varphi : G \longrightarrow G$ given by $\varphi(a) = a^{-1}$ is an isomorphism if and only if $G$ is abelian.

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**6.** (15 pts.)

a) In the group $G = GL(2, \mathbb{R})$ of invertible $2 \times 2$ matrices with real entries show that

$$H = \left\{ \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} \in GL(2, \mathbb{R}) : b \in \mathbb{R} \right\}$$

is a subgroup of $G$.

b) Show that $H$ is isomorphic to the group $\mathbb{R}$ of all real numbers, under addition.

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