Exam

Name___________________________________

**SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.**

**Solve the problem.**

1) Show that if a particle's velocity vector is always orthogonal to its acceleration vector then the particle's speed is constant. 1) ______________

2) Prove that $\int_a^b (\mathbf{r}_1(t) + \mathbf{r}_2(t))\,dt = \int_a^b \mathbf{r}_1(t)\,dt + \int_a^b \mathbf{r}_2(t)\,dt$ 2) ______________

**Provide an appropriate response.**

3) Derive the equations 3) ______________

$$x = x_o + (v_o \cos \alpha)t,$$

$$y = y_o + (v_o \sin \alpha)t - \frac{1}{2}gt^2$$

by solving the following initial value problem for a vector **r** in the plane.

Differential equation: $\dfrac{d^2\mathbf{r}}{dt^2} = -g\mathbf{j}$

Initial conditions: $\mathbf{r}(0) = x_o\mathbf{i} + y_o\mathbf{j}$

$\dfrac{d\mathbf{r}}{dt}(0) = v_o \cos \alpha)\mathbf{i} + (v_o \sin \alpha)\mathbf{j}$

**Solve the problem.**

4) The angle between two intersecting planes is defined to be the (acute) angle determined by the normal vectors. 4) ______________

(a) If $\mathbf{n}_1$ and $\mathbf{n}_2$ are the normals to two planes, show that the angle between the planes is

$$\theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}\right).$$

(b) Find the angle between the planes $-4x + 3y + 6z = 5$ and $-2x + 3y + 5z = 5$.

5) $\mathbf{r}(t) = (t^2 - 4)\mathbf{i} + (2t - 4)\mathbf{j} + 3\mathbf{k}$ 5) ______________

Find(a) **T**, **N**, and **B**, (b) curvature, (c) torsion and (d) tangential and normal components of acceleration.

**Provide an appropriate response.**

6) Derive the equations 6) ______________

$$x = x_o + \frac{v_o}{k}\left(1 - e^{-kt}\right)\cos\alpha,$$

$$y = y_o + \frac{v_o}{k}\left(1 - e^{-kt}\right)\sin\alpha + \frac{g}{k^2}\left(1 - kt - e^{-kt}\right)$$

by solving the following initial value problem for a vector **r** in the plane.

Differential equation: $\frac{d^2\mathbf{r}}{dt^2} = -g\mathbf{j} - k\mathbf{v} = -g\mathbf{j} - k\frac{d\mathbf{r}}{dt}$

Initial conditions: $\mathbf{r}(0) = x_o\mathbf{i} + y_o\mathbf{j}$

$\frac{d\mathbf{r}}{dt}(0) = \mathbf{v}_o = \left(v_o\cos\alpha\right)\mathbf{i} + \left(v_o\sin\alpha\right)\mathbf{j}$

The **drag coefficient** k is a positive constant representing resistance due to air density, $v_o$ and $\alpha$ are the projectile's initial speed and launch angle, and g is the acceleration of gravity.

7) A baseball is hit when it is 3.4 feet above the ground. It leaves the bat with an initial velocity of 140 ft/s at a launch angle of 20°. At the instant the ball is hit, an instantaneous gust of wind blows against the ball, adding a component of $-10\mathbf{i}$ ft/s to the ball's initial velocity. Find the range and flight time of the ball, assuming that the ball is not caught. 7) ______________

**Solve the problem.**

8) $\mathbf{r}(t) = (\ln(\cos t) + 5)\mathbf{i} + 4\mathbf{j} + (7 + t)\mathbf{k}$ 8) ______________

Find(a) **T**, **N**, and **B**, (b) curvature, (c) torsion and (d) tangential and normal components of acceleration.

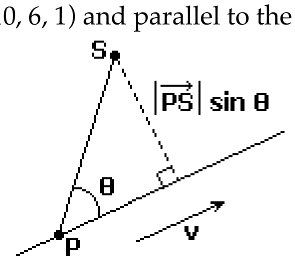
9) Follow these steps to find the distance from a point S to a line that passes through a point P parallel to a vector v as shown in the figure. 9) ______________

(a) Show that the length of the component of $\overrightarrow{PS}$ normal to the line is $\left|\overrightarrow{PS}\right| \sin\theta$.

(b) Show that the distance d from S to the line through P parallel to **v** is

$$d = \frac{\left|\overrightarrow{PS} \times \mathbf{v}\right|}{|\mathbf{v}|}.$$

(c) Find the distance from the point S(8, –9, –4) and the line passing through the point P(10, 6, 1) and parallel to the vector $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j} + 12\mathbf{k}$.




10) $\mathbf{r}(t) = (3t\sin t + 3\cos t)\mathbf{i} + (3t\cos t - 3\sin t)\mathbf{j} - 4\mathbf{k}$ 10) ______________

Find(a) **T**, **N**, and **B**, (b) curvature, (c) torsion and (d) tangential and normal components of acceleration.

11) (a) Show that the curve $\mathbf{r}(t) = (1 - \cos(t))\mathbf{i} + \cos(t)\mathbf{j} + \sin(t)\mathbf{k}$, $0 \le t \le 2\pi$, is an ellipse by showing that it is the intersection of a right circular cylinder and a plane. Find equations for the cylinder and plane. 11) ______________

(b) Sketch the ellipse on the cylinder. Add to your sketch the unit tangent vectors at $t = 0$ and $t = \frac{\pi}{2}$.

(c) Show that the acceleration vector always lies parallel to the plane (orthogonal to a vector normal to the plane). Thus, if you draw the acceleration as a vector attached to the ellipse, it will lie in the plane of the ellipse. Add the acceleration vectors for $t = 0$ and $t = \frac{\pi}{2}$ to your sketch.

(d) Write an integral for the length of the ellipse. Do not try to evaluate the integral – it is nonelementary.

12) $\mathbf{r}(t) = \cosh(t)\mathbf{i} + \sinh(t)\mathbf{j} + \mathbf{k}$ 12) ______________

Find(a) **T**, **N**, and **B**, (b) curvature, (c) torsion and (d) tangential and normal components of acceleration.

**Provide an appropriate response.**

13) A golf ball leaves the ground at a 33° angle at a speed of 90 ft/s. Will it clear the top of a 29 ft tree that is in the way, 135 ft down the fairway? Explain. 13) ______________

**Solve the problem.**

14) $\mathbf{r}(t) = \left(4 + 6\sin\left(\frac{2}{3}t\right)\right)\mathbf{i} + \left(10 + 6\cos\left(\frac{2}{3}t\right)\right)\mathbf{j} + 3t\mathbf{k}$ 14) ______________

Find(a) **T**, **N**, and **B**, (b) curvature, (c) torsion and (d) tangential and normal components of acceleration.

15) $\mathbf{r}(t) = (3 + t)\mathbf{i} + (9 + \ln(\sec t))\mathbf{j} - 5\mathbf{k}$ 15) ______________

Find(a) **T**, **N**, and **B**, (b) curvature, (c) torsion and (d) tangential and normal components of acceleration.

16) Prove that if **u** is a differentiable function of t and f is a differentiable scalar function of t, then $\frac{d}{dt}(f\mathbf{u}) = \frac{df}{dt}\mathbf{u} + f\frac{d\mathbf{u}}{dt}$. 16) ______________

**Provide an appropriate response.**

17) A human cannonball is to be fired with an initial speed of 90 ft/s. The circus performer hopes to land on a cushion located 200 feet downrange at the same height as the muzzle of the cannon. The circus is being held in a large room with a flat ceiling 35 feet higher than the muzzle. Can the performer be fired to the cushion without striking the ceiling? If so, what is the proper firing angle? ($g = 32$ ft/s$^2$) 17) ______________

**Solve the problem.**

18) Prove that if **v** and **u** are differentiable functions of t, then $\frac{d}{dt}(\mathbf{u} + \mathbf{v}) = \frac{d\mathbf{u}}{dt} + \frac{d\mathbf{v}}{dt}$. 18) ______________

19) Prove that $\int_a^b k\mathbf{r}(t)\,dt = k\int_a^b \mathbf{r}(t)\,dt$ for any scalar constant k. 19) ____________

**MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**

**Find the arc length parameter along the curve from the point where t = 0 by evaluating $s = \int_0^t |\mathbf{v}(\tau)|\,d\tau$.**

20) $\mathbf{r}(t) = (2\cos t)\mathbf{i} + (2\sin t)\mathbf{j} + 5t\mathbf{k}$ 20) ______

A) $\sqrt{29}t$ B) $3\sqrt{6}t$ C) $\sqrt{33}t$ D) $\frac{\sqrt{29}}{2}t$

**Find the torsion of the space curve.**

21) $\mathbf{r}(t) = (7t\sin t + 7\cos t)\mathbf{i} + (7t\cos t) - 7\sin t)\mathbf{j} - 5\mathbf{k}$ 21) ______

A) $\tau = 0$ B) $\tau = 1$ C) Undefined D) $\tau = -1$

**The position vector of a particle is r(t). Find the requested vector.**

22) The velocity at $t = \frac{\pi}{4}$ for $\mathbf{r}(t) = 4\sec^2(t)\mathbf{i} - 6\tan(t)\mathbf{j} + 7t^2\mathbf{k}$ 22) ______

A) $\mathbf{v}\left(\frac{\pi}{4}\right) = 16\mathbf{i} - 12\mathbf{j} + \frac{7}{2}\pi\mathbf{k}$ B) $\mathbf{v}\left(\frac{\pi}{4}\right) = -12\mathbf{j} + \frac{7}{2}\pi\mathbf{k}$

C) $\mathbf{v}\left(\frac{\pi}{4}\right) = 16\mathbf{i} + 12\mathbf{j} - \frac{7}{2}\pi\mathbf{k}$ D) $\mathbf{v}\left(\frac{\pi}{4}\right) = -12\mathbf{j} - \frac{7}{2}\pi\mathbf{k}$

**Find the principal unit normal vector N for the curve r(t).**

23) $\mathbf{r}(t) = (t^2 + 2)\mathbf{j} + (2t - 6)\mathbf{k}$ 23) ______

A) $\mathbf{N}(t) = -\frac{1}{\sqrt{t^2+1}}\mathbf{j} + \frac{t}{\sqrt{t^2+1}}\mathbf{k}$ B) $\mathbf{N}(t) = \frac{1}{\sqrt{t^2+1}}\mathbf{j} - \frac{t}{\sqrt{t^2+1}}\mathbf{k}$

C) $\mathbf{N}(t) = \frac{1}{\sqrt{(t^2+1)^3}}\mathbf{j} - \frac{t}{\sqrt{(t^2+1)^3}}\mathbf{k}$ D) $\mathbf{N}(t) = -\frac{1}{\sqrt{(t^2+1)^3}}\mathbf{j} + \frac{t}{\sqrt{(t^2+1)^3}}\mathbf{k}$

**Find the torsion of the space curve.**

24) $\mathbf{r}(t) = \left(7 + 2\sin 2t\right)\mathbf{i} + 3t\mathbf{j} + \left(5 + 2\cos 2t\right)\mathbf{k}$ 24) ______

A) $\tau = \frac{6}{5}$ B) $\tau = -\frac{6}{5}$ C) $\tau = \frac{12}{5}$ D) $\tau = \frac{6}{25}$

**Find the curvature of the space curve.**

25) $\mathbf{r}(t) = \frac{8}{9}(1 + t)^{3/2}\mathbf{i} + \frac{8}{9}(1 - t)^{3/2}\mathbf{j} + \frac{2}{3}t\mathbf{k}$ 25) ______

A) $\kappa = \frac{1}{6}\sqrt{\frac{2}{1 - t^2}}$ B) $\kappa = \frac{1}{3}\sqrt{\frac{2}{1 + t^2}}$ C) $\kappa = \frac{1}{3}\sqrt{\frac{2}{1 - t^2}}$ D) $\kappa = \frac{1}{6}\sqrt{\frac{2}{1 + t^2}}$

**Solve the problem.**

26) The following equations each describe the motion of a particle. For which path is the particle's velocity vector always orthogonal to its acceleration vector? 26) _______

(1) $\mathbf{r}(t) = t^4\mathbf{i} + t^3\mathbf{j}$

(2) $\mathbf{r}(t) = \cos(2t)\mathbf{i} + \sin(4t)\mathbf{j}$

(3) $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j}$

(4) $\mathbf{r}(t) = \cos(3t)\mathbf{i} + \sin(3t)\mathbf{j}$

A) Path (4)
B) Path (1)
C) Path (2) and Path (4)
D) Path (3)

**Evaluate the integral.**

27) $\int_0^1 \left[4t\mathbf{i} + 6t^2\mathbf{j} - \frac{12}{(1+t)^4}\mathbf{k}\right] dt$ 27) _______

A) $2\mathbf{i} + 2\mathbf{j} - \frac{7}{2}\mathbf{k}$
B) $4\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}$
C) $2\mathbf{i} - 2\mathbf{j} - \frac{7}{2}\mathbf{k}$
D) $4\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$

**Find T, N, and B for the given space curve.**

28) $\mathbf{r}(t) = (\ln(\cos t) + 6)\mathbf{i} + 8\mathbf{j} + (5 + t)\mathbf{k}$, $-\pi/2 < t < \pi/2$ 28) _______

A) $\mathbf{T} = (\sin t)\mathbf{i} + (\cos t)\mathbf{k}$; $\mathbf{N} = (\cos t)\mathbf{i} - (\sin t)\mathbf{k}$; $\mathbf{B} = -\mathbf{j}$

B) $\mathbf{T} = (\sin t)\mathbf{i} + (\cos t)\mathbf{k}$; $\mathbf{N} = (\cos t)\mathbf{i} - (\sin t)\mathbf{k}$; $\mathbf{B} = \mathbf{j}$

C) $\mathbf{T} = (-\sin t)\mathbf{i} + (\cos t)\mathbf{k}$; $\mathbf{N} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{k}$; $\mathbf{B} = -\mathbf{j}$

D) $\mathbf{T} = (-\sin t)\mathbf{i} + (\cos t)\mathbf{k}$; $\mathbf{N} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{k}$; $\mathbf{B} = \mathbf{j}$

**Solve the problem. Assume projectile is ideal, launch angle is measured from the horizontal, and launch is over a horizontal surface, unless stated otherwise.**

29) A baseball is hit when it is 2.5 ft above the ground. It leaves the bat with an initial velocity of 140 ft/sec at a launch angle of 20°. At the instant the ball is hit, an instantaneous gust of wind blows against the ball, adding a component of $-11.8\mathbf{i}$ (ft/sec) to the ball's initial velocity. How high does the baseball go? Round your answer to the nearest tenth. 29) _______

A) 47.9 ft
B) 38.3 ft
C) 35.8 ft
D) 40.3 ft

**Solve the problem.**

30) A particle of mass m moves along the ellipse $\mathbf{r}(t) - (4\cos t)\mathbf{i} + (4\sin t)\mathbf{j}$. Find the force on the particle at the point where $t = \frac{\pi}{6}$. 30) _______

A) $\mathbf{F}\left(\frac{\pi}{6}\right) = 2m\mathbf{i} - 2\sqrt{3}m\mathbf{j}$
B) $\mathbf{F}\left(\frac{\pi}{6}\right) = -2\sqrt{2}m\mathbf{i} - 2\sqrt{2}m\mathbf{j}$
C) $\mathbf{F}\left(\frac{\pi}{6}\right) = -2m\mathbf{i} - 2\sqrt{3}m\mathbf{j}$
D) $\mathbf{F}\left(\frac{\pi}{6}\right) = -2\sqrt{2}m\mathbf{i} + 2\sqrt{2}m\mathbf{j}$

**Find the curvature of the space curve.**

31) $\mathbf{r}(t) = t\mathbf{i} + (\sinh t)\mathbf{j} + (\cosh t)\mathbf{k}$ 31) _______

A) $\kappa = \cosh^2 t$
B) $\kappa = \frac{\tanh^2 t}{2}$
C) $\kappa = \frac{\text{sech}^2 t}{2}$
D) $\kappa = \sinh^2 t$

**Find the torsion of the space curve.**

32) $\mathbf{r}(t) = (2t - 10)\mathbf{i} + (t^2 - 6)\mathbf{j} + 10\mathbf{k}$ 32) _______

A) $\tau = -1$
B) $\tau = 1$
C) Undefined
D) $\tau = 0$

**The position vector of a particle is r(t). Find the requested vector.**

33) The velocity at t = 0 for $\mathbf{r}(t) = \ln(t^3 - 10t^2 + 5)\mathbf{i} - \sqrt{t^2 + 9}\mathbf{j} - 2\cos(t)\mathbf{k}$ 33) _______

A) $\mathbf{v}(0) = \frac{1}{5}\mathbf{i} - \frac{1}{6}\mathbf{j} + 2\mathbf{k}$
B) $\mathbf{v}(0) = -\frac{1}{3}\mathbf{j}$
C) $\mathbf{v}(0) = \frac{1}{3}\mathbf{j}$
D) $\mathbf{v}(0) = \frac{1}{5}\mathbf{i} - \frac{1}{3}\mathbf{j}$

**Calculate the arc length of the indicated portion of the curve r(t).**

34) $\mathbf{r}(t) = 4t\mathbf{i} + \left(5\cos\frac{3}{5}t\right)\mathbf{j} + \left(5\sin\frac{3}{5}t\right)\mathbf{k}$; $-1 \le t \le 10$ 34) _______

A) 45 B) 225 C) 55 D) 275

**Solve the problem.**

35) The orbit of *Skylab 4* had a period of T = 93.11 minutes. Calculate the semimajor axis of the orbit. (Earth's mass = $5.975 \times 10^{24}$ kg and G = $6.6720 \times 10^{-11}$ $\mathrm{Nm^2kg^{-2}}$). 35) _______

A) 444 km B) $6.805 \times 10^6$ km C) $4.440 \times 10^4$ km D) 6805 km

**Find the arc length parameter along the curve from the point where t = 0 by evaluating $s = \int_0^t |\mathbf{v}(\tau)|\, d\tau$.**

36) $\mathbf{r}(t) = (3 + 2t)\mathbf{i} + (5 + 3t)\mathbf{j} + (3 - 6t)\mathbf{k}$ 36) _______

A) 8t B) 5t C) 7t D) 9t

**Find the curvature of the curve r(t).**

37) $\mathbf{r}(t) = (10 + \ln(\sec t))\mathbf{i} + (3 + t)\mathbf{k}$, $-\pi/2 < t < \pi/2$ 37) _______

A) $\kappa = -\cos t$ B) $\kappa = \cos t$ C) $\kappa = 1 - \cos t$ D) $\kappa = \sin t$

**The vector r(t) is the position vector of a particle in space at time t. Find the time or times in the given time interval when the velocity and acceleration vectors are orthogonal.**

38) $\mathbf{r}(t) = (t - 5\sin t)\mathbf{i} + (7 - 5\cos t)\mathbf{j}$; $0 \le t \le 2\pi$ 38) _______

A) $t = \pi/2, 3\pi/2$
B) $0 \le t \le 2\pi$
C) $t = 0, \pi/2, \pi, 3\pi/2, 2\pi$
D) $t = 0, \pi, 2\pi$

**Find the curvature of the space curve.**

39) $\mathbf{r}(t) = (t + 2)\mathbf{i} + 5\mathbf{j} + (\ln(\sec t) + 5)\mathbf{k}$ 39) _______

A) $\kappa = \sin t$ B) $\kappa = \sec t$ C) $\kappa = \cos t$ D) $\kappa = \csc t$

**Find T, N, and B for the given space curve.**

40) $\mathbf{r}(t) = (5t\sin t + 5\cos t)\mathbf{i} + (5t\cos t - 5\sin t)\mathbf{j} - 9\mathbf{k}$ 40) _______

A) $\mathbf{T} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j}$; $\mathbf{N} = \sin(t)\mathbf{i} - (\cos t)\mathbf{j}$; $\mathbf{B} = 9\mathbf{k}$
B) $\mathbf{T} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j}$; $\mathbf{N} = (\sin t)\mathbf{i} - (\cos t)\mathbf{j}$; $\mathbf{B} = \mathbf{k}$
C) $\mathbf{T} = (-\cos t)\mathbf{i} + (\sin t)\mathbf{j}$; $\mathbf{N} = (\sin t)\mathbf{i} + (\cos t)\mathbf{j}$; $\mathbf{B} = -\mathbf{k}$
D) $\mathbf{T} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j}$; $\mathbf{N} = (\sin t)\mathbf{i} - (\cos t)\mathbf{j}$; $\mathbf{B} = -\mathbf{k}$

**The position vector of a particle is r(t). Find the requested vector.**

41) The velocity at t = 0 for $\mathbf{r}(t) = \cos(2t)\mathbf{i} + 9\ln(t - 3)\mathbf{j} - \frac{t^3}{3}\mathbf{k}$ 41) _______

A) $\mathbf{v}(0) = 3\mathbf{j}$ B) $\mathbf{v}(0) = 2\mathbf{i} - 3\mathbf{j}$ C) $\mathbf{v}(0) = -2\mathbf{i} - 3\mathbf{j}$ D) $\mathbf{v}(0) = -3\mathbf{j}$

**Solve the problem.**

42) Intelsat 5 was launched in December 1980 with a mass of 1928 kg. Its orbital period is 1417.67 minutes. Find the semimajor axis of the orbit. ($G = 6.6720 \times 10^{-11}$ $Nm^2kg^{-2}$, mass of earth = $5.975 \times 10^{24}$ kg). 42) _______

A) $4.181 \times 10^7$ km B) $2.728 \times 10^6$ km C) 2728 km D) 41,810 km

**Find the length of the indicated portion of the curve.**

43) $\mathbf{r}(t) = (5 + 2t)\mathbf{i} + (4 + 3t)\mathbf{j} + (3 - 6t)\mathbf{k}$, $-1 \le t \le 0$ 43) _______

A) 7 B) 8 C) 9 D) 5

**Find the principal unit normal vector N for the curve r(t).**

44) $\mathbf{r}(t) = (9 + t)\mathbf{i} + (2 + \ln(\cos t))\mathbf{k}$, $-\pi/2 < t < \pi/2$ 44) _______

A) $\mathbf{N}(t) = (-\cos t)\mathbf{i} - (\ln(\cos t))\mathbf{k}$ B) $\mathbf{N}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{k}$

C) $\mathbf{N}(t) = (-\sin t)\mathbf{i} - (\cos t)\mathbf{k}$ D) $\mathbf{N}(t) = (-\cos t)\mathbf{i} - (\tan t)\mathbf{k}$

**Solve the problem.**

45) The orbit of *Vanguard 1* had a semimajor axis of a = 8872 km. Calculate the period of the satellite. (Earth's mass = $5.975 \times 10^{24}$ kg and $G = 6.6720 \times 10^{-11}$ $Nm^2kg^{-2}$). 45) _______

A) 12 hr B) 138.6 min C) 78.2 min D) 1.98 hr

**Solve the problem. Assume projectile is ideal, launch angle is measured from the horizontal, and launch is over a horizontal surface, unless stated otherwise.**

46) An ideal projectile is launched from level ground at a launch angle of $26^\circ$ and an initial speed of 48 m/sec. How far away from the launch point does the projectile hit the ground? 46) _______

A) ≈ 60 m B) ≈ 185 m C) ≈ 230 m D) ≈ 290 m

**Find the curvature of the space curve.**

47) $\mathbf{r}(t) = -7\mathbf{i} + (t + 2)\mathbf{j} + (\ln(\cos t) + 10)\mathbf{k}$ 47) _______

A) $\kappa = \cos t$ B) $\kappa = \sec t$ C) $\kappa = \csc t$ D) $\kappa = \sin t$

**Find the unit tangent vector of the given curve.**

48) $\mathbf{r}(t) = 12t^7\mathbf{i} - 4t^7\mathbf{j} + 3t^7\mathbf{k}$ 48) _______

A) $\mathbf{T}(t) = \frac{12}{13}\mathbf{i} + \frac{4}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}$ B) $\mathbf{T}(t) = \frac{12}{169}\mathbf{i} - \frac{4}{169}\mathbf{j} + \frac{3}{169}\mathbf{k}$

C) $\mathbf{T}(t) = \frac{12}{13}\mathbf{i} - \frac{4}{13}\mathbf{j} - \frac{3}{13}\mathbf{k}\frac{4}{13}$ D) $\mathbf{T}(t) = \frac{12}{13}\mathbf{i} - \frac{4}{13}\mathbf{j} + \frac{3}{13}\mathbf{k}$

**Calculate the arc length of the indicated portion of the curve r(t).**

49) $\mathbf{r}(t) = 2t^4\mathbf{i} + 11t^4\mathbf{j} + 10t^4\mathbf{k}$, $-1 \le t \le 3$ 49) _______

A) 300 B) 4500 C) 18,000 D) 1200

50) $\mathbf{r}(t) = (7 - 2t)\mathbf{i} + (5 + 6t)\mathbf{j} + (9t - 7)\mathbf{k}$, $-10 \le t \le -3$ 50) _______

A) 77 B) $-1573$ C) 847 D) $-143$

**Solve the problem. Assume projectile is ideal, launch angle is measured from the horizontal, and launch is over a horizontal surface, unless stated otherwise.**

51) A baseball is hit when it is 2.5 ft above the ground. It leaves the bat with an initial velocity of 147 ft/sec at a launch angle of 24°. At the instant the ball is hit, an instantaneous gust of wind blows against the ball, adding a component of $-8.1\mathbf{i}$ (ft/sec) to the ball's initial velocity. Find a vector equation for the path of the baseball. 51) _______

A) $\mathbf{r} = ((147 \cos 24)t - 8.1)\mathbf{i} + (2.5 + (147 \sin 24t - 16t^2)\mathbf{j}$

B) $\mathbf{r} = (147 \cos 24 - 8.1)t\mathbf{i} + (2.5 + (147 \sin 24)t - 16t^2\mathbf{j}$

C) $\mathbf{r} = (147 \cos 24 - 8.1)t\mathbf{i} + (2.5 + (147 \sin 24)t + 16t^2\mathbf{j}$

D) $\mathbf{r} = (147 \sin 24 - 8.1)t\mathbf{i} + (2.5 + (147 \cos 24)t - 16t^2)\mathbf{j}$

**Solve the initial value problem.**

52) Differential Equation: $\dfrac{d^2\mathbf{r}}{dt^2} = 9t^2\mathbf{i} - t\mathbf{j}$ 52) _______

Initial Conditions: $\left.\dfrac{d\mathbf{r}}{dt}\right|_{r=0} = -2\mathbf{i}$, $\mathbf{r}(0) = 10\mathbf{i} - 3\mathbf{k}$

A) $\mathbf{r}(t) = \left(\dfrac{3}{4}t^4 + 2t + 10\right)\mathbf{i} + \dfrac{t^3}{6}\mathbf{j} - 3\mathbf{k}$

B) $\mathbf{r}(t) = \left(\dfrac{9}{10}t^4 - 2t + 10\right)\mathbf{i} - \dfrac{t^3}{6}\mathbf{j} - 3\mathbf{k}$

C) $\mathbf{r}(t) = \left(\dfrac{3}{4}t^4 - 2t + 10\right)\mathbf{i} - \dfrac{t^3}{6}\mathbf{j} - 3\mathbf{k}$

D) $\mathbf{r}(t) = \left(\dfrac{3}{4}t^4 - 2t + 10\right)\mathbf{i} - \dfrac{t^3}{2}\mathbf{j} - 3\mathbf{k}$

**If r(t) is the position vector of a particle in the plane at time t, find the indicated vector.**

53) $\mathbf{r}(t) = (6 \cos t)\mathbf{i} + (4 \sin t)\mathbf{j}$ 53) _______

Find the acceleration vector.

A) $\mathbf{a}(t) = (6 \cos t)\mathbf{i} + (4 \sin t)\mathbf{j}$

B) $\mathbf{a}(t) = (-6 \sin t)\mathbf{i} + (-4 \cos t)\mathbf{j}$

C) $\mathbf{a}(t) = (-6 \cos t)\mathbf{i} + (-4 \sin t)\mathbf{j}$

D) $\mathbf{a}(t) = (6 \sin t)\mathbf{i} + (4 \cos t)\mathbf{j}$

**The position vector of a particle is r(t). Find the requested vector.**

54) The acceleration at $t = \dfrac{\pi}{12}$ for $\mathbf{r}(t) = (7t - 3t^3)\mathbf{i} + 3\tan(3t)\mathbf{j} + e^{4t}\mathbf{k}$ 54) _______

A) $\mathbf{a}\left(\dfrac{\pi}{12}\right) = -\dfrac{3}{2}\pi\mathbf{i} - 108\mathbf{j} + 16e^{1/3\pi}\mathbf{k}$

B) $\mathbf{a}\left(\dfrac{\pi}{12}\right) = -\dfrac{3}{2}\pi\mathbf{i} - 108\mathbf{j} + 16\mathbf{k}$

C) $\mathbf{a}\left(\dfrac{\pi}{12}\right) = \dfrac{3}{2}\pi\mathbf{i} + 108\mathbf{j} + 16e^{1/3\pi}\mathbf{k}$

D) $\mathbf{a}\left(\dfrac{\pi}{12}\right) = -\dfrac{3}{2}\pi\mathbf{i} + 108\mathbf{j} + 16e^{1/3\pi}\mathbf{k}$

**Evaluate the integral.**

55) $\displaystyle\int_0^4 \left[-\mathbf{i} - \frac{2t}{(3 + t^2)^2}\mathbf{j} + 6\sqrt{t}\mathbf{k}\right] dt$ 55) _______

A) $-4\mathbf{i} - \dfrac{16}{57}\mathbf{j} + 32\mathbf{k}$

B) $-4\mathbf{i} + \dfrac{4}{21}\mathbf{j} + 32\mathbf{k}$

C) $-4\mathbf{i} - \dfrac{16}{57}\mathbf{j} - 32\mathbf{k}$

D) $+4\mathbf{i} + \dfrac{16}{57}\mathbf{j} + 32\mathbf{k}$

**If r(t) is the position vector of a particle in the plane at time t, find the indicated vector.**

56) $\mathbf{r}(t) = (4\ln(3t))\mathbf{i} + (2t^3)\mathbf{j}$ 56) _______

Find the acceleration vector.

A) $\mathbf{a}(t) = -\frac{4}{t^2}\mathbf{i} + 12t\mathbf{j}$

B) $\mathbf{a}(t) = \frac{4}{t^2}t^{-2}\mathbf{i} + 12t\mathbf{j}$

C) $\mathbf{a}(t) = \frac{4}{t}\mathbf{i} + 6t\mathbf{j}$

D) $\mathbf{a}(t) = -\frac{4}{3}t^{-2}\mathbf{i} + 12t\mathbf{j}$

**Solve the problem.**

57) At time t = 0, a particle is located at the point (4, 4, 5). It travels in a straight line to the point (8, 16, 8), has speed 5 at (4, 4, 5) and constant acceleration $4\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}$. Find an equation for the position vector $\mathbf{r}(t)$ of the particle at time t. 57) _______

A) $\mathbf{r}(t) = \left(\frac{t^2}{2} + \frac{5}{11}t\right)(4\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}) + 4\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$

B) $\mathbf{r}(t) = \left(t^2 + 5t\right)(4\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}) + 4\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$

C) $\mathbf{r}(t) = \left(\frac{t^2}{2} + \frac{5}{169}t\right)(4\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}) + 4\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}$

D) $\mathbf{r}(t) = \left(t^2 + \frac{5}{11}t\right)(4\mathbf{i} + 12\mathbf{j} + 3\mathbf{k}) + 4\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$

**For the smooth curve r(t), find the parametric equations for the line that is tangent to r at the given parameter value $t = t_0$.**

58) $\mathbf{r}(t) = (3\sin t)\mathbf{i} - (5\cos 3t)\mathbf{j} + e^{-7t}\mathbf{k}$ ; $t_o = 0$ 58) _______

A) x = 3t, y = −5, z = 1 − 7t

B) x = 3t, y = 5, z = 1 + t

C) x = 3t, y = −5, z = 1 − t

D) x = 3, y = −5t, z = −7 + t

**Solve the initial value problem.**

59) Differential Equation: $\frac{d\mathbf{r}}{dt} = -3t\mathbf{i} + 9t\mathbf{j} + 9t\mathbf{k}$ 59) _______

Initial Condition: $\mathbf{r}(0) = -2\mathbf{i} + 4\mathbf{k}$

A) $\mathbf{r}(t) = (-3t^2 - 4)\mathbf{i} + 9t^2\mathbf{j} + (9t^2 + 8)\mathbf{k}$

B) $\mathbf{r}(t) = \frac{-3t^2 - 4}{2}\mathbf{i} + \frac{9}{2}t^2\mathbf{j} + \frac{9t^2 + 8}{2}\mathbf{k}$

C) $\mathbf{r}(t) = \frac{-3t^2 - 2}{2}\mathbf{i} + \frac{9}{2}t^2\mathbf{j} + \frac{9t^2 + 4}{2}\mathbf{k}$

D) $\mathbf{r}(t) = \frac{-3t^2 - 2}{2}\mathbf{i} + \frac{9}{2}t^2\mathbf{j} - \frac{9t^2 + 4}{2}\mathbf{k}$

**Solve the problem.**

60) At what times in the interval $0 \le t \le \pi$ are the velocity and the acceleration vectors of the motion $\mathbf{r}(t)$ $10\mathbf{i} + 4\cos(t)\mathbf{j} + 3\sin(t)\mathbf{k}$ orthogonal? 60) _______

A) $t = \frac{\pi}{2}$

B) Velocity and acceleration vectors are orthogonal for all t in $0 \le t \le \pi$.

C) $t = 0$; $t = \frac{\pi}{2}$; $t = \pi$

D) t = 0

**The vector r(t) is the position vector of a particle at time t. Find the angle between the velocity and the acceleration vectors at time t = 0.**

61) $\mathbf{r}(t) = \sin^{-1}(5t)\mathbf{i} + \ln(4t^2 + 1)\mathbf{j} + \sqrt{7t^2 + 1}\mathbf{k}$ 61) _______

A) 0 B) $\frac{\pi}{4}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{3}$

**For the smooth curve r(t), find the parametric equations for the line that is tangent to r at the given parameter value $t = t_0$.**

62) $\mathbf{r}(t) = (2t^2 - 3t)\mathbf{i} + (t + 7)\mathbf{j} + \mathbf{k}$ ; $t_0 = 2$ 62) _______

A) $x = 2 + 5t, y = 9 + t, z = 0$
B) $x = 5t, y = t, z = t$
C) $x = 2 + 5t, y = 9 + t, z = 1$
D) $x = 2 + t, y = t, z = t$

**If r(t) is the position vector of a particle in the plane at time t, find the indicated vector.**

63) $\mathbf{r}(t) = (\cot t)\mathbf{i} + (\csc t)\mathbf{j}$ 63) _______
Find the velocity vector.

A) $\mathbf{v}(t) = (\sec^2 t)\mathbf{i} + (\tan t \sec t)\mathbf{j}$
B) $\mathbf{v}(t) = (-\sec^2 t)\mathbf{i} - (\tan t \sec t)\mathbf{j}$
C) $\mathbf{v}(t) = (-\csc^2 t)\mathbf{i} - (\cot t \csc t)\mathbf{j}$
D) $\mathbf{v}(t) = (\csc^2 t)\mathbf{i} + (\cot t \csc t)\mathbf{j}$

**Solve the problem. Assume projectile is ideal, launch angle is measured from the horizontal, and launch is over a horizontal surface, unless stated otherwise.**

64) Find the muzzle speed of a gun whose maximum range is 16.6 km. Round answer to the nearest tenth. 64) _______

A) 13.3 km/sec B) 163.2 km/sec C) 12.8 km/sec D) 162.7 km/sec

**Calculate the arc length of the indicated portion of the curve r(t).**

65) Following the curve $\mathbf{r}(t) = 5t\mathbf{i} + (12 \cos t)\mathbf{j} + (12 \sin t)\mathbf{k}$ in the direction of increasing of arc length, find the point that lies $65\pi$ units away from the point where $t = 0$. 65) _______

A) $(25\pi, 0, 0)$ B) $(25\pi, -12, 0)$ C) $(25, -12, 0)$ D) $(25\pi, 0, -12)$

**The position vector of a particle is r(t). Find the requested vector.**

66) The velocity at $t = 2$ for $\mathbf{r}(t) = (9 - 4t^2)\mathbf{i} + (7t + 4)\mathbf{j} - e^{-7t}\mathbf{k}$ 66) _______

A) $\mathbf{v}(2) = 16\mathbf{i} + 7\mathbf{j} + 7e^{-14}\mathbf{k}$
B) $\mathbf{v}(2) = -16\mathbf{i} + 7\mathbf{j} - 7e^{-14}\mathbf{k}$
C) $\mathbf{v}(2) = -16\mathbf{i} + 7\mathbf{j} + 7e^{-14}\mathbf{k}$
D) $\mathbf{v}(2) = -8\mathbf{i} + 7\mathbf{j} + 7e^{-14}\mathbf{k}$

**Find the length of the indicated portion of the curve.**

67) $\mathbf{r}(t) = (1 + 3t)\mathbf{i} + (1 + 4t)\mathbf{j} + (5 - 5t)\mathbf{k}$, $-1 \le t \le 0$ 67) _______

A) $\sqrt{41}$ B) $\sqrt{34}$ C) 5 D) $5\sqrt{2}$

**Solve the initial value problem.**

68) Differential Equation: $\frac{d\mathbf{r}}{dt} = (t^4 + 2t^2)\mathbf{i} + 4t\mathbf{j}$ 68) _______

Initial Condition: $\mathbf{r}(0) = -5\mathbf{i} + 3\mathbf{j}$

A) $\mathbf{r}(t) = \left(\frac{t^5}{5} + \frac{2t^3}{3} + 3\right)\mathbf{i} + (2t^2 - 5)\mathbf{j}$
B) $\mathbf{r}(t) = \left(\frac{t^5}{5} + \frac{2t^3}{3} - 5\right)\mathbf{i} + (t^2 + 3)\mathbf{j}$
C) $\mathbf{r}(t) = \left(\frac{t^5}{5} + \frac{2t^3}{3} - 5\right)\mathbf{i} + (2t^2 + 3)\mathbf{j}$
D) $\mathbf{r}(t) = \left(\frac{t^5}{5} + \frac{2t^3}{3}\right)\mathbf{i} + 2t^2\mathbf{j}$

**The position vector of a particle is r(t). Find the requested vector.**

69) The acceleration at t = 1 for $\mathbf{r}(t) = (8t - 2t^4)\mathbf{i} + (2 - t)\mathbf{j} + (4t^2 - 2t)\mathbf{k}$ 69) _______

A) $\mathbf{a}(1) = 24\mathbf{i} + 8\mathbf{k}$ B) $\mathbf{a}(1) = -24\mathbf{i} - \mathbf{j} + 8\mathbf{k}$

C) $\mathbf{a}(1) = -6\mathbf{i} + 8\mathbf{k}$ D) $\mathbf{a}(1) = -24\mathbf{i} + 8\mathbf{k}$

**Solve the problem.**

70) A particle moves along a cycloid in the xy-plane in such a way that its position at time t is $\mathbf{r}(t) = (3t + \cos 3t)\mathbf{i} + (\sin 3t - 1)\mathbf{j}$. Find the maximum and minimum values of $|\mathbf{v}|$ and $|\mathbf{a}|$. (*Hint*: Find the extreme values of $|\mathbf{v}|^2$ and $|\mathbf{a}|^2$ first and take square roots later). 70) _______

A) $|\mathbf{v}|_{min} = 0; |\mathbf{v}|_{max} = 6; |\mathbf{a}|_{min} = |\mathbf{a}|_{max} = 9$

B) $|\mathbf{v}|_{min} = |\mathbf{v}|_{max} = 0; |\mathbf{a}|_{min} = -9; |\mathbf{a}|_{max} = 9$

C) $|\mathbf{v}|_{min} = 0; |\mathbf{v}|_{max} = 6; |\mathbf{a}|_{min} = |\mathbf{a}|_{max} = 3$

D) $|\mathbf{v}|_{min} = 3; |\mathbf{v}|_{max} = 3; |\mathbf{a}|_{min} = -9; |\mathbf{a}|_{max} = 9$

**Find the torsion of the space curve.**

71) $\mathbf{r}(t) = (t - 1)\mathbf{i} + (\ln(\sec t) + 7)\mathbf{j} - 3\mathbf{k}, -\pi/2 < t < \pi/2$ 71) _______

A) Undefined B) $\tau = 1$ C) $\tau = 0$ D) $\tau = -1$

**For the curve r(t), write the acceleration in the form $a_T\mathbf{T} + a_N\mathbf{N}$.**

72) $\mathbf{r}(t) = 4(1 + t)^{3/2}\mathbf{i} + 4(1 - t)^{3/2}\mathbf{j} + 3t\mathbf{k}$ 72) _______

A) $\mathbf{a} = 3\sqrt{\dfrac{2}{1 - t^2}}\mathbf{T}$ B) $\mathbf{a} = \dfrac{1}{3}\sqrt{\dfrac{1}{1 - t^2}}\mathbf{N}$

C) $\mathbf{a} = \mathbf{T} + 3\sqrt{\dfrac{2}{1 - t^2}}\mathbf{N}$ D) $\mathbf{a} = 3\sqrt{\dfrac{2}{1 - t^2}}\mathbf{N}$

**Solve the problem.**

73) The orbit of *Sputnik 1* had a semimajor axis of a = 6955 km. Calculate the period of the satellite. (Earth's mass = $5.975 \times 10^{24}$ kg and G = $6.6720 \times 10^{-11}$ $Nm^2kg^{-2}$). 73) _______

A) 0.18 sec B) 9250 hr C) 124 min D) 96.2 min

**Find the unit tangent vector of the given curve.**

74) $\mathbf{r}(t) = (6t \cos t - 6 \sin t)\mathbf{j} + (6t \sin t + 6 \cos t)\mathbf{k}$ 74) _______

A) $\mathbf{T}(t) = (-\sin t)\mathbf{j} + (\cos t)\mathbf{k}$ B) $\mathbf{T}(t) = (6 \cos t)\mathbf{j} - (6 \sin t)\mathbf{k}$

C) $\mathbf{T}(t) = -\dfrac{1}{6}(\sin t)\mathbf{j} + \dfrac{1}{6}(\cos t)\mathbf{k}$ D) $\mathbf{T}(t) = (-6 \sin t)\mathbf{j} + (6 \cos t)\mathbf{k}$

75) $\mathbf{r}(t) = (8 \sin^3 5t)\mathbf{i} + (8 \cos^3 5t)\mathbf{j}$ 75) _______

A) $\mathbf{T}(t) = (\sin 5t)\mathbf{i} - (\cos 5t)\mathbf{j}$ B) $\mathbf{T}(t) = (8 \sin 5t)\mathbf{i} - (8 \cos 5t)\mathbf{j}$

C) $\mathbf{T}(t) = (120 \sin 5t)\mathbf{i} - (120 \cos 5t)\mathbf{j}$ D) $\mathbf{T}(t) = (8 \cos 5t)\mathbf{i} - (8 \sin 5t)\mathbf{j}$

**Solve the problem. Assume projectile is ideal, launch angle is measured from the horizontal, and launch is over a horizontal surface, unless stated otherwise.**

76) An athlete puts a 16–lb shot at an angle of 41° to the horizontal from 6.2 ft above the ground at an initial speed of 32 ft/sec. How far forward does the shot travel before it hits the ground? Round answer to the nearest tenth. 76) _______

A) 6 ft B) 1.6 ft C) 110.2 ft D) 37.7 ft

**Solve the initial value problem.**

77) Differential Equation: $\frac{d\mathbf{r}}{dt} = (7 \sec 7t \tan 7t)\ \mathbf{i} + \frac{t}{(t^2+3)^2}\mathbf{j} + t^2\mathbf{k}$ 77) _______

Initial Condition: $\mathbf{r}(0) = 8\mathbf{i} + \frac{17}{6}\mathbf{j} - 8\mathbf{k}$

A) $\mathbf{r}(t) = (\csc 7t + 7)\mathbf{i} + \left(3 - \frac{1}{2t^2+6}\right)\mathbf{j} + \left(\frac{t^3}{3} - 8\right)\mathbf{k}$

B) $\mathbf{r}(t) = (\csc 7t + 7)\mathbf{i} + \left(3 - \frac{1}{2t^2+6}\right)\mathbf{j} + (t^3 - 8)\mathbf{k}$

C) $\mathbf{r}(t) = (\sec 7t + 7)\mathbf{i} + \left(3 - \frac{1}{2t^2+6}\right)\mathbf{j} + \left(\frac{t^3}{3} - 8\right)\mathbf{k}$

D) $\mathbf{r}(t) = (\sec 7t + 7)\mathbf{i} - \frac{1}{2t^2+6}\mathbf{j} + \left(\frac{t^3}{3} - 8\right)\mathbf{k}$

**The vector r(t) is the position vector of a particle in space at time t. Find the time or times in the given time interval when the velocity and acceleration vectors are orthogonal.**

78) $\mathbf{r}(t) = 5 \cos t\mathbf{i} + 7 \sin t\mathbf{j};\ 0 \le t \le 2\pi$ 78) _______

A) $0 \le t \le 2\pi$ B) $t = 0, \pi, 2\pi$

C) $t = 0, \pi/2, \pi, 3\pi/2, 2\pi$ D) $t = \pi/2, 3\pi/2$

**Find T, N, and B for the given space curve.**

79) $\mathbf{r}(t) = (\cosh t)\mathbf{i} + (\sinh t)\mathbf{j} + \mathbf{k}$ 79) _______

A) $\mathbf{T} = -\frac{\sqrt{2}}{2}(\tanh t \operatorname{sech} t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$; $\mathbf{N} = (-\operatorname{sech} t)\mathbf{i} - (\tanh t)\mathbf{k}$;

$\mathbf{B} = \frac{\sqrt{2}}{2}(\sinh t)\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j} - \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$

B) $\mathbf{T} = \frac{\sqrt{2}}{2}(\tanh t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$; $\mathbf{N} = (\operatorname{sech} t)\mathbf{i} - (\tanh t)\mathbf{k}$;

$\mathbf{B} = -\frac{\sqrt{2}}{2}(\sinh t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} - \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$

C) $\mathbf{T} = -\frac{\sqrt{2}}{2}(\tanh t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$; $\mathbf{N} = (-\operatorname{sech} t)\mathbf{i} - (\sinh t)\mathbf{k}$ ;

$\mathbf{B} = -\frac{\sqrt{2}}{2}(\sinh t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} - \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$

D) $\mathbf{T} = \frac{\sqrt{2}}{2}(\tanh t \operatorname{sech} t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$; $\mathbf{N} = (\operatorname{sech}^2 t)\mathbf{i} - (\sinh t)\mathbf{k}$;

$\mathbf{B} = \frac{\sqrt{2}}{2}(\sinh t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} + \frac{\sqrt{2}}{2}(\operatorname{sech} t)\mathbf{k}$

**For the curve r(t), find an equation for the indicated plane at the given value of t.**

80) $\mathbf{r}(t) = (t^2 - 8)\mathbf{i} + (2t - 6)\mathbf{j} + 9\mathbf{k}$; osculating plane at $t = 6$. 80) _______

A) $z = -9$
B) $x + y + (z - 9) = 0$
C) $z = 9$
D) $x + y + (z + 9) = 0$

**Evaluate the integral.**

81) $\int_1^2 ((1 - 3t)\mathbf{i} + 5\sqrt{t}\,\mathbf{j})\,dt$ 81) _______

A) $-\frac{7}{2}\mathbf{i} + \frac{10}{3}(2\sqrt{2} - 1)\,\mathbf{j}$
B) $-\frac{7}{2} + \frac{10}{3}(2\sqrt{2} - 1)$
C) $-\frac{9}{2}\mathbf{i} + \frac{10}{3}(2\sqrt{2} - 1)\,\mathbf{j}$
D) $5\frac{\sqrt{2} - 2}{4}\,\mathbf{j}$

**If r(t) is the position vector of a particle in the plane at time t, find the indicated vector.**

82) $\mathbf{r}(t) = (-5t^2 - 9)\mathbf{i} + \left(\frac{1}{15}t^3\right)\mathbf{j}$. 82) _______

Find the velocity vector of the particle.

A) $\mathbf{v}(t) = (-10t)\mathbf{i} + \left(\frac{1}{5}t^3\right)\mathbf{j}$
B) $\mathbf{v}(t) = \left(\frac{1}{5}t^2\right)\mathbf{i} + (-10t)\mathbf{j}$
C) $\mathbf{v}(t) = (-10)\mathbf{i} + \left(\frac{2}{5}t\right)\mathbf{j}$
D) $\mathbf{v}(t) = (-10t)\mathbf{i} - \left(\frac{1}{5}t^3\right)\mathbf{j}$

**Find the curvature of the curve r(t).**

83) $\mathbf{r}(t) = (3 + \cos 4t - \sin 4t)\mathbf{i} + (7 + \sin 4t + \cos 4t)\mathbf{j} + 3\mathbf{k}$ 83) _______

A) $\kappa = \sqrt{2}$
B) $\kappa = 2$
C) $\kappa = \frac{\sqrt{2}}{2}$
D) $\kappa = 2\sqrt{2}$

**For the curve r(t), find an equation for the indicated plane at the given value of t.**

84) $\mathbf{r}(t) = (6 \sin 2t + 10)\mathbf{i} + (6 \cos 13t) - 3)\mathbf{j} + 5t\mathbf{k}$; osculating plane at $t = 2.5\pi$. 84) _______

A) $\frac{5}{13}(x - 10) + 6\left(z - \frac{25}{2}\pi\right) = 0$
B) $\frac{5}{169}(y - 3) + \frac{6}{13}\left(z - \frac{25}{2}\pi\right) = 0$
C) $\frac{5}{13}(y + 3) + 6\left(z + \frac{25}{2}\right) = 0$
D) $\frac{5}{13}(y + 3) + 6\left(z - \frac{25}{2}\pi\right) = 0$

**Find the length of the indicated portion of the curve.**

85) $\mathbf{r}(t) = (4\cos t)\mathbf{i} + (4\sin t)\mathbf{j} + 7t\mathbf{k}$, $0 \le t \le \pi/2$ 85) _______

A) $\frac{9t}{2}\pi$
B) $\frac{\sqrt{11}t}{2}\pi$
C) $\frac{\sqrt{65}t}{2}\pi$
D) $\frac{\sqrt{65}t}{4}\pi$

**Provide an appropriate response.**

86) A baseball is hit when it is 3.0 feet above the ground. It leaves the bat with an initial speed of 154 ft/s, making an angle of 20° with the horizontal. Assuming a drag coefficient $k = 0.13$, how high does the ball go, and when does it reach maximum height? 86) _______

For projectiles with linear drag:

$$x = x_o + \frac{v_o}{k}\left(1 - e^{-kt}\right)\cos\alpha,$$

$$y = y_o + \frac{v_o}{k}\left(1 - e^{-kt}\right)\sin\alpha + \frac{g}{k^2}\left(1 - kt - e^{-kt}\right)$$

where k is the drag coefficient, $v_o$ and $\alpha$ are the projectile's initial speed and launch angle, and g is the acceleration of gravity (32 ft/$s^2$).

A) Maximum height = 0.04 feet; time to maximum height = –0.2 s
B) Maximum height = 3.34 feet; time to maximum height = 0.03 s
C) Maximum height = 8.54 feet; time to maximum height = 0.75 s
D) Maximum height = 3.29 feet; time to maximum height = 0.07 s

**The position vector of a particle is r(t). Find the requested vector.**

87) The velocity at $t = 1$ for $\mathbf{r}(t) = (4t^2 + 3t + 4)\mathbf{i} - 2t^3\mathbf{j} + (3 - t^2)\mathbf{k}$ 87) _______

A) $\mathbf{v}(1) = 11\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$
B) $\mathbf{v}(1) = 11\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$
C) $\mathbf{v}(1) = 7\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}$
D) $\mathbf{v}(1) = 5\mathbf{i} - 6\mathbf{j} - 2\mathbf{k}$

**Find the curvature of the curve r(t).**

88) $\mathbf{r}(t) = (1 + 5\cos 9)t\ \mathbf{i} - (9 + 5\sin 9t)\mathbf{j} + 10\mathbf{k}$ 88) _______

A) $\kappa = \frac{9}{5}$
B) $\kappa = 5$
C) $\kappa = \frac{1}{25}$
D) $\kappa = \frac{1}{5}$

**Calculate the arc length of the indicated portion of the curve r(t).**

89) $\mathbf{r}(t) = (9\cos^3 6t)\mathbf{j} + (9\sin^3 6t)\mathbf{k};\ \frac{1}{2}\pi \le t \le \frac{7}{12}\pi$ 89) _______

A) 27
B) $\frac{27}{2}$
C) $\frac{9}{2}$
D) 0

**Evaluate the integral.**

90) $\displaystyle\int_{-\pi/4}^{\pi/4} (4\cos t\ \mathbf{i} + 5\sin t\ \mathbf{j})\,dt$ 90) _______

A) $5\sqrt{2}\ \mathbf{i}$
B) $4\sqrt{2}\ \mathbf{i}$
C) 0
D) $4\sqrt{2}\ \mathbf{i} + 5\sqrt{2}\ \mathbf{j}$

**Find the principal unit normal vector N for the curve r(t).**

91) $\mathbf{r}(t) = (4\sin t)\mathbf{i} + (4\cos t)\mathbf{j} + 7\mathbf{k}$ 91) _______

A) $\mathbf{N}(t) = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j}$
B) $\mathbf{N}(t) = (-\sin t)\mathbf{i} + (\cos t)\mathbf{j}$
C) $\mathbf{N}(t) = (-\sin t)\mathbf{i} - (\cos t)\mathbf{j}$
D) $\mathbf{N}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j}$

**Solve the initial value problem.**

92) Differential Equation: $\dfrac{d^2\mathbf{r}}{dt^2} = (7t - 8)\mathbf{i}$ 92) _______

Initial Conditions: $\left.\dfrac{d\mathbf{r}}{dt}\right|_{r=0} = -\mathbf{k}$, $\mathbf{r}(0) = 8\mathbf{i} + 3\mathbf{j} + 9\mathbf{k}$

A) $\mathbf{r}(t) = \left(\frac{7}{3}t^3 - 8t^2 + 8\right)\mathbf{i} + 3\mathbf{j} + (9 - t)\mathbf{k}$

B) $\mathbf{r}(t) = \left(\frac{7}{3}t^3 + 4t^2 + 8\right)\mathbf{i} - 3\mathbf{j} + (t - 9)\mathbf{k}$

C) $\mathbf{r}(t) = \left(\frac{7}{6}t^3 - 4t^2 + 8\right)\mathbf{i} + 3\mathbf{j} + (9 - t)\mathbf{k}$

D) $\mathbf{r}(t) = \left(\frac{7}{6}t^3 + 4t^2 + 8\right)\mathbf{i} - 3\mathbf{j} + (9 - t)\mathbf{k}$

93) Differential Equation: $\dfrac{d\mathbf{r}}{dt} = \dfrac{7}{2}(t + 5)^{5/2}\mathbf{i} + e^t\mathbf{j}$ 93) _______

Initial Condition: $\mathbf{r}(0) = 0$

A) $\mathbf{r}(t) = [(t + 5)^{7/2} - 5^{7/2}]\mathbf{i} + (e^t - 1)\mathbf{j}$

B) $\mathbf{r}(t) = [(t + 5)^{7/2} + 5^{7/2}]\mathbf{i} + (e^t + 1)\mathbf{j}$

C) $\mathbf{r}(t) = (t + 5)^{7/2}\mathbf{i} + e^t\mathbf{j}$

D) $\mathbf{r}(t) = [(t + 5)^{9/2} - 5^{9/2}]\mathbf{i} + (e^t - 1)\mathbf{j}$

**For the curve r(t), find an equation for the indicated plane at the given value of t.**

94) $\mathbf{r}(t) = (t + 2)\mathbf{i} + (\ln(\cos t) - 6)\mathbf{j} + 5\mathbf{k}$, $-\pi/2 < t < \pi.2$; rectifying plane at $t = \frac{\pi}{4}$. 94) _______

A) $\frac{\sqrt{2}}{2}(x - (1 + 2)) = 0$

B) $\frac{\sqrt{2}}{2}\left(x - \left(\frac{\pi}{4} + 2\right)\right) - \frac{\sqrt{2}}{2}\left(y - \left(\ln\left(\frac{\sqrt{2}}{2}\right) - 6\right)\right) = 0$

C) $\frac{\sqrt{2}}{2}\left(x - \left(\frac{\pi}{4} + 2\right)\right) + \frac{\sqrt{2}}{2}\left(y - \left(\ln\left(\frac{\sqrt{2}}{2}\right) - 6\right)\right) = 0$

D) $-\frac{\sqrt{2}}{2}\left(x - \left(\frac{\pi}{4} + 2\right)\right) + \frac{\sqrt{2}}{2}\left(y - \left(\ln\left(\frac{\sqrt{2}}{2}\right) - 6\right)\right) = 0$

**Solve the problem.**

95) Two asteroids orbit a star. The orbit of the first asteroid has a minimum radius $r_0 = 2.30 \times 10^6$ km and an eccentricity e = 0.20. The second asteroid has a minimum radius $r_0 = 2.45 \times 10^6$ km and an eccentricity e = 0.00. Determine if the orbits cross one another. If so, find the angles at which the two orbits intersect. 95) _______

A) $\theta = 50.75°$

B) The orbits do not cross.

C) $\theta = 71.56°$

D) $\theta = 50.75°$ and $\theta = 309.25°$

**Find the arc length parameter along the curve from the point where t = 0 by evaluating $s = \int_0^t v(\tau)\, d\tau$.**

96) $\mathbf{r}(t) = (e^t\cos t)\mathbf{i} + (e^t\sin t)\mathbf{j} + 6e^t\mathbf{k}$ 96) _______

A) $\sqrt{38}te^t - \sqrt{38}$

B) $\sqrt{34}e^t - \sqrt{34}$

C) $\sqrt{34}e^t - \sqrt{38}$

D) $\sqrt{38}e^t - \sqrt{34}$

**Solve the problem. Assume projectile is ideal, launch angle is measured from the horizontal, and launch is over a horizontal surface, unless stated otherwise.**

97) A sprung gun at ground level fires a tennis ball at an angle of 58°. The ball lands 11 m away. What was the ball's initial speed? Round answer to the nearest tenth. 97) _______

A) 11.3 m/sec B) 11.0 m/sec C) 119.9 m/sec D) 3.5 m/sec

**The vector r(t) is the position vector of a particle at time t. Find the angle between the velocity and the acceleration vectors at time t = 0.**

98) $\mathbf{r}(t) = (6t^2 + 10)\mathbf{i} + (5t^3 - 4t)\mathbf{k}$ 98) _______

A) $\frac{\pi}{2}$ B) $\pi$ C) 0 D) $\frac{\pi}{4}$

**Solve the problem.**

99) At perihelion, Jupiter's velocity $v_o$ is $1.371 \times 10^4$ m/s. Its orbit has an eccentricity e = 0.0484. Find the minimum orbital radius $r_o$ for Jupiter. ($G = 6.6720 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$, mass of sun = $1.99 \times 10^{30}$ kg). 99) _______

A) $7.064 \times 10^8$ km B) $2.721 \times 10^4$ km C) $1.015 \times 10^{13}$ km D) $7.406 \times 10^8$ km

**For the curve r(t), write the acceleration in the form $a_T\mathbf{T} + a_N\mathbf{N}$.**

100) $\mathbf{r}(t) = (8t \sin t + 8 \cos t)\mathbf{i} + (8t \cos t - 8 \sin t)\mathbf{j} + 5\mathbf{k}$ 100) _______

A) $\mathbf{a} = \frac{1}{8t}\mathbf{N}$ B) $\mathbf{a} = 8\mathbf{T} + 8t\mathbf{N}$ C) $\mathbf{a} = 8\mathbf{T} + \frac{1}{8t}\mathbf{N}$ D) $\mathbf{a} = 8t\mathbf{N}$

**The vector r(t) is the position vector of a particle at time t. Find the angle between the velocity and the acceleration vectors at time t = 0.**

101) $\mathbf{r}(t) = e^{7t}\mathbf{i} + (9 + e^{-7t})\mathbf{j} + (5 \cos 3t)\mathbf{k}$ 101) _______

A) $\frac{\pi}{2}$ B) $\frac{\pi}{4}$ C) 0 D) $\frac{\pi}{3}$

**Find T, N, and B for the given space curve.**

102) $\mathbf{r}(t) = \left(1 + 4 \sin \frac{3}{4}t\right)\mathbf{i} + \left(7 + 4 \cos \frac{3}{4}t\right)\mathbf{j} + 4t\mathbf{k}$ 102) _______

A) $\mathbf{T} = \frac{3}{5}(\sin 0.75t)\mathbf{i} - \frac{3}{5}(\cos 0.75t)\mathbf{j}$ ; $\mathbf{N} = (-\sin 0.75t)\mathbf{i} - (\cos 0.75t)\mathbf{j}$ ; $\mathbf{B} = \frac{4}{5}(\cos 0.75t)\mathbf{i} - \frac{4}{5}(\sin 0.75t)\mathbf{j} - \frac{3}{5}\mathbf{k}$

B) $\mathbf{T} = \frac{3}{5}(\cos 0.75t)\mathbf{i} - \frac{3}{5}(\sin 0.75t)\mathbf{j} + \frac{4}{5}\mathbf{k}$ ; $\mathbf{N} = (-\sin 0.75t)\mathbf{i} - (\cos 0.75t)\mathbf{j}$ ; $\mathbf{B} = -\frac{3}{5}\mathbf{k}$

C) $\mathbf{T} = \frac{3}{5}(\cos 0.75t)\mathbf{i} - \frac{3}{5}(\sin 0.75t)\mathbf{j}$ ; $\mathbf{N} = (-\sin 0.75t)\mathbf{i} - (\cos 0.75t)\mathbf{j}$ ; $\mathbf{B} = \frac{4}{5}(\cos 0.75t)\mathbf{i} - \frac{4}{5}(\sin 0.75t)\mathbf{j} - \frac{3}{5}\mathbf{k}$

D) $\mathbf{T} = \frac{3}{5}(\cos 0.75t)\mathbf{i} - \frac{3}{5}(\sin 0.75t)\mathbf{j} + \frac{4}{5}\mathbf{k}$ ; $\mathbf{N} = (-\sin 0.75t)\mathbf{i} - (\cos 0.75t)\mathbf{j}$ ; $\mathbf{B} = \frac{4}{5}(\cos 0.75t)\mathbf{i} - \frac{4}{5}(\sin 0.75t)\mathbf{j} - \frac{3}{5}\mathbf{k}$

**For the curve r(t), write the acceleration in the form $a_T\mathbf{T} + a_N\mathbf{N}$.**

103) $\mathbf{r}(t) = (t + 7)\mathbf{i} + (\ln(\sec t) - 6)\mathbf{j} + 10\mathbf{k}$, $-\pi/2 < t < \pi/2$ 103) ______

A) $\mathbf{a} = (\cos t)\mathbf{T} + (\cos t)\mathbf{N}$
B) $\mathbf{a} = (\csc t)\mathbf{T} + (\sec t)\mathbf{N}$
C) $\mathbf{a} = (\sec t \tan t)\mathbf{T} + (\sec t)\mathbf{N}$
D) $\mathbf{a} = (\sec^2 t)\mathbf{T} + (\cos t)\mathbf{N}$

**Evaluate the integral.**

104) $\displaystyle\int_0^{\frac{\pi}{4}} [(7\sec^2 t)\mathbf{i} - (5 + \sin t)\mathbf{j} - (9\sec t \tan t)\mathbf{k}]dt$ 104) ______

A) $7\mathbf{i} + \left(\dfrac{2\sqrt{2} - 5\pi - 4}{2}\right)\mathbf{j} + 9(1 + \sqrt{2})\mathbf{k}$
B) $7\mathbf{i} + \left(\dfrac{2\sqrt{2} - 5\pi - 4}{4}\right)\mathbf{j} + 9(1 - \sqrt{2})\mathbf{k}$
C) $7\mathbf{i} + \left(\dfrac{2\sqrt{2} - 5\pi - 4}{2}\right)\mathbf{j} + 9(1 - \sqrt{2})\mathbf{k}$
D) $7\mathbf{i} + \left(\dfrac{2\sqrt{2} + 5\pi + 4}{2}\right)\mathbf{j} + 9(1 + \sqrt{2})\mathbf{k}$

**Find the curvature of the space curve.**

105) $\mathbf{r}(t) = -8\mathbf{i} + (8 + 2t)\mathbf{j} + (t^2 + 8)\mathbf{k}$ 105) ______

A) $\kappa = \dfrac{1}{2\sqrt{t^2 + 1}}$
B) $\kappa = \dfrac{1}{2(t^2 + 1)^{3/2}}$
C) $\kappa = \dfrac{1}{(t^2 + 1)^{3/2}}$
D) $\kappa = -\dfrac{1}{2(t^2 + 1)^{3/2}}$

**Find the curvature of the curve r(t).**

106) $\mathbf{r}(t) = (9t + 3)\mathbf{i} - 10\mathbf{j} + (9 - \frac{9}{2}t^2)\mathbf{k}$ 106) ______

A) $\kappa = 9\sqrt{1 + t^2}$
B) $\kappa = \dfrac{1}{9\left(1 + t^2\right)^{3/2}}$
C) $\kappa = 9\left(1 + t^2\right)^{3/2}$
D) $\kappa = \dfrac{1}{9\sqrt{1 + t^2}}$

**For the curve r(t), find an equation for the indicated plane at the given value of t.**

107) $\mathbf{r}(t) = (\cosh t)\mathbf{i} + (\sinh t)\mathbf{j} + t\mathbf{k}$; rectifying plane at $t = 0$. 107) ______

A) $x - y = 1$
B) $x = -1$
C) $x - y = -1$
D) $x = 1$

**Solve the initial value problem.**

108) Differential Equation: $\dfrac{d\mathbf{r}}{dt} = (27t^2 - 2)\mathbf{i} - \mathbf{j} + \dfrac{1}{\sqrt{1 + t}}\mathbf{k}$ 108) ______

Initial Condition: $\mathbf{r}(0) = -7\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$

A) $\mathbf{r}(t) = (9t^3 + 2t + 7)\mathbf{i} + (3 + t)\mathbf{j} + (\sqrt{1 + t} + 6)\mathbf{k}$
B) $\mathbf{r}(t) = (9t^3 - 2t - 7)\mathbf{i} + (3 - t)\mathbf{j} + (\sqrt{1 + t} - 6)\mathbf{k}$
C) $\mathbf{r}(t) = (27t^3 - 2t - 7)\mathbf{i} + (3 - t)\mathbf{j} + (\sqrt{1 + t} + 6)\mathbf{k}$
D) $\mathbf{r}(t) = (9t^3 - 2t - 7\mathbf{i} + (3 - t)\mathbf{j} + (2\sqrt{1 + t} + 6)\mathbf{k}$

**Find T, N, and B for the given space curve.**

109) $\mathbf{r}(t) = \frac{20}{9}(1 + t)^{3/2}\mathbf{i} + \frac{20}{9}(1 - t)^{3/2}\mathbf{j} + \frac{5}{3}t\mathbf{k}$ 109) ______

A) $\mathbf{T} = \frac{2}{3}\sqrt{1 + t}\mathbf{i} + \frac{2}{3}\sqrt{1 - t}\mathbf{j} + \frac{1}{3}\mathbf{k}$; $\mathbf{N} = \frac{1}{2}\sqrt{2 - 2t}\mathbf{i} + \frac{1}{2}\sqrt{2 + 2t}\mathbf{j}$; $\mathbf{B} = \frac{1}{6}\sqrt{2 + 2t}\mathbf{i} + \frac{1}{6}\sqrt{2 - 2t}\mathbf{j} + \frac{2\sqrt{2}}{3}\mathbf{k}$

B) $\mathbf{T} = \frac{1}{3}\sqrt{1 + t}\mathbf{i} - \frac{1}{3}\sqrt{1 - t}\mathbf{j} + \frac{1}{3}\mathbf{k}$; $\mathbf{N} = \frac{1}{2}\sqrt{2 - 2t}\mathbf{i} + \frac{1}{2}\sqrt{2 + 2t}\mathbf{j}$; $\mathbf{B} = -\frac{1}{6}\sqrt{2 + 2t}\mathbf{i} + \frac{1}{6}\sqrt{2 - 2t}\mathbf{j} + \frac{\sqrt{2}}{3}\mathbf{k}$

C) $\mathbf{T} = \frac{2}{3}\sqrt{1 + t}\mathbf{i} - \frac{2}{3}\sqrt{1 - t}\mathbf{j} + \frac{1}{3}\mathbf{k}$; $\mathbf{N} = \frac{1}{2}\sqrt{2 - 2t}\mathbf{i} + \frac{1}{2}\sqrt{2 + 2t}\mathbf{j}$ ; $\mathbf{B} = -\frac{1}{6}\sqrt{2 + 2t}\mathbf{i} + \frac{1}{6}\sqrt{2 - 2t}\mathbf{j} + \frac{2\sqrt{2}}{3}\mathbf{k}$

D) $\mathbf{T} = \frac{10}{3}\sqrt{1 + t}\mathbf{i} - \frac{10}{3}\sqrt{1 - t}\mathbf{j} + \frac{5}{3}\mathbf{k}$; $\mathbf{N} = \frac{5}{2}\sqrt{2 - 2t}\mathbf{i} + \frac{5}{2}\sqrt{2 + 2t}\mathbf{j}$; $\mathbf{B} = -\frac{5}{6}\sqrt{2 + 2t}\mathbf{i} + \frac{5}{6}\sqrt{2 - 2t}\mathbf{j} + \frac{10}{3}\sqrt{2}\mathbf{k}$

**Solve the problem. Assume projectile is ideal, launch angle is measured from the horizontal, and launch is over a horizontal surface, unless stated otherwise.**

110) A fan in the bleachers at Wrigley Field throws an opposing player's home run back onto the playing field. Assume that the fan is 30 feet above the field and that the ball is launched at an angle of 38°. When will the ball hit the ground if its initial speed is 22 ft/sec? Round answer to the nearest tenth. 110) ______

A) 2.0 sec B) 4.2 sec C) 1.9 sec D) 1.0 sec

**Find the arc length parameter along the curve from the point where t = 0 by evaluating** $s = \int_0^t v(\tau)\, d\tau$.

111) $\mathbf{r}(t) = (1 + 5t)\mathbf{i} + (1 + 8t)\mathbf{j} + (4 - 4t)\mathbf{k}$ 111) ______

A) $\sqrt{105}t$ B) $4\sqrt{5}t$ C) $\sqrt{89}t$ D) $\sqrt{41}t$

**Provide an appropriate response.**

112) Increasing the initial speed of a projectile by a factor of 2 increases its range by what factor? Assume the elevation is the same. 112) ______

A) Factor of 1.41

B) Factor of 4

C) The elevation must be specified before an answer can be found.

D) Factor of 2

**Find the principal unit normal vector N for the curve r(t).**

113) $\mathbf{r}(t) = (3t \sin t + 3 \cos t)\mathbf{i} + (3t \cos t - 3 \sin t))\mathbf{k}$ 113) ______

A) $\mathbf{N}(t) = (\cos t)\mathbf{i} - (\sin t)\mathbf{k}$ B) $\mathbf{N}(t) = (-\sin t)\mathbf{i} - (\cos t)\mathbf{k}$

C) $\mathbf{N}(t) = -\frac{\sqrt{2}}{2}(\cos t)\mathbf{i} - \frac{\sqrt{2}}{2}(\sin t)\mathbf{k}$ D) $\mathbf{N}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{k}$

**Evaluate the integral.**

114) $\int_0^1 \left[\frac{1}{\sqrt{2t - t^2}}\mathbf{i} + te^{3t}\mathbf{j} - \frac{12t^2}{(7 + 4t^3)^2}\mathbf{k}\right] dt$ 114) ______

A) $\frac{2e^3 + 1}{9}\mathbf{j} + \frac{4}{35}\mathbf{k}$

B) $-\frac{3}{2}\pi\mathbf{i} + \frac{2e^3 + 1}{9}\mathbf{j} + \frac{4}{7}\mathbf{k}$

C) $-\frac{3}{2}\pi\mathbf{i} + \frac{2e^3}{9}\mathbf{j} + \frac{4}{35}\mathbf{k}$

D) $\frac{\pi}{2}\mathbf{i} + \frac{2e^3 + 1}{9}\mathbf{j} - \frac{4}{77}\mathbf{k}$

**Find the unit tangent vector of the given curve.**

115) $\mathbf{r}(t) = (5 + 2t^7)\mathbf{i} + (4 + 10t^7)\mathbf{j} + (8 + 11t^7)\mathbf{k}$ 115) ______

A) $\mathbf{T}(t) = \frac{2}{15}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{11}{15}\mathbf{k}$

B) $\mathbf{T}(t) = \frac{2}{225}\mathbf{i} + \frac{2}{45}\mathbf{j} + \frac{11}{225}\mathbf{k}$

C) $\mathbf{T}(t) = 2\mathbf{i} + 10\mathbf{j} + 11\mathbf{k}$

D) $\mathbf{T}(t) = \frac{14}{15}\mathbf{i} + \frac{14}{3}\mathbf{j} + \frac{77}{15}\mathbf{k}$

**Find T, N, and B for the given space curve.**

116) $\mathbf{r}(t) = (t^2 - 7)\mathbf{i} + (2t - 10)\mathbf{j} + 5\mathbf{k}$ 116) ______

A) $\mathbf{T} = \frac{t}{t^2 + 1}\mathbf{i} + \frac{1}{t^2 + 1}\mathbf{j}$; $\mathbf{N} = \frac{1}{t^2 + 1}\mathbf{i} - \frac{t}{t^2 + 1}\mathbf{j}$; $\mathbf{B} = -\mathbf{k}$

B) $\mathbf{T} = \frac{t}{t^2 + 1}\mathbf{i} + \frac{1}{t^2 + 1}\mathbf{j}$; $\mathbf{N} = \frac{1}{t^2 + 1}\mathbf{i} - \frac{t}{t^2 + 1}\mathbf{j}$; $\mathbf{B} = \mathbf{k}$

C) $\mathbf{T} = \frac{t}{\sqrt{t^2 + 1}}\mathbf{i} + \frac{1}{\sqrt{t^2 + 1}}\mathbf{j}$; $\mathbf{N} = \frac{1}{\sqrt{t^2 + 1}}\mathbf{i} - \frac{t}{\sqrt{t^2 + 1}}\mathbf{j}$; $\mathbf{B} = -\mathbf{k}$

D) $\mathbf{T} = \frac{t}{t^2 + 1}\mathbf{i} - \frac{1}{t^2 + 1}\mathbf{j}$; $\mathbf{N} = -\frac{1}{t^2 + 1}\mathbf{i} - \frac{t}{t^2 + 1}\mathbf{j}$; $\mathbf{B} = -\mathbf{k}$

**Evaluate the integral.**

117) $\int_0^3 \left[\left(\frac{4}{\sqrt{1 + t}}\right)\mathbf{i} - \left(3t^2\right)\mathbf{j} + \left(\frac{10t}{(1 + t^2)^2}\right)\mathbf{k}\right] dt$ 117) ______

A) $8\mathbf{i} - 27\mathbf{j} + \frac{9}{2}\mathbf{k}$

B) $4\mathbf{i} - 27\mathbf{j} + \frac{9}{2}\mathbf{k}$

C) $8\mathbf{i} - 27\mathbf{j} + \frac{9}{10}\mathbf{k}$

D) $8\mathbf{i} + 27\mathbf{j} + \frac{9}{10}\mathbf{k}$

**Solve the problem. Assume projectile is ideal, launch angle is measured from the horizontal, and launch is over a horizontal surface, unless stated otherwise.**

118) An ideal projectile is launched from the origin at an angle of $\alpha$ radians to the horizontal and an initial speed of 75 ft/sec. Find the position function $\mathbf{r}(t)$ for this projectile. 118) ______

A) $\mathbf{r}(t) = (75t \cos \alpha)\mathbf{i} + (75t \sin \alpha - 16t^2)\mathbf{j}$

B) $\mathbf{r}(t) = (75t \sin \alpha - 16t^2)\mathbf{i} + (75t \cos \alpha)\mathbf{j}$

C) $\mathbf{r}(t) = (75t \sin \alpha)\mathbf{i} + (75t \cos \alpha - 16t^2)\mathbf{j}$

D) $\mathbf{r}(t) = (75t \cos \alpha - 32t^2)\mathbf{i} + (75t \sin \alpha)\mathbf{j}$

**Solve the problem.**

119) The following equations each describe the motion of a particle. For which path is the particle's speed constant? 119) ______

(1) $\mathbf{r}(t) = t^2\mathbf{i} + t^4\mathbf{j}$
(2) $\mathbf{r}(t) = \cos(6t)\mathbf{i} + \sin(8t)\mathbf{j}$
(3) $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j}$
(4) $\mathbf{r}(t) = \cos(5t^2)\mathbf{i} + \sin(5t^2)\mathbf{j}$

A) Path (1)
B) Path (3)
C) Path (2) and Path (3)
D) Path (4) and Path (2)

**Find the curvature of the space curve.**

120) $\mathbf{r}(t) = 12t\mathbf{i} + \left(1 + 7\cos\frac{5}{7}t\right)\mathbf{j} + \left(6 + 7\sin\frac{5}{7}\right)\mathbf{k}$ 120) ______

A) $\kappa = \frac{25}{91}$
B) $\kappa = \frac{25}{1183}$
C) $\kappa = \frac{5}{169}$
D) $\kappa = \frac{5}{13}$

**Find the unit tangent vector of the given curve.**

121) $\mathbf{r}(t) = (3 - 2t)\mathbf{i} + (2t - 2)\mathbf{j} + (10 + t)\mathbf{k}$ 121) ______

A) $\mathbf{T}(t) = -\frac{2}{9}\mathbf{i} + \frac{2}{9}\mathbf{j} + \frac{1}{9}\mathbf{k}$
B) $\mathbf{T}(t) = \frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$
C) $\mathbf{T}(t) = \frac{2}{9}\mathbf{i} - \frac{2}{9}\mathbf{j} - \frac{1}{9}\mathbf{k}$
D) $\mathbf{T}(t) = -\frac{2}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}$

**Solve the initial value problem.**

122) Differential Equation: $\frac{d\mathbf{r}}{dt} = (-\cos t)\mathbf{i} + (4t^3 - 4)\mathbf{j}$ 122) ______

Initial Condition: $\mathbf{r}(0) = 4\mathbf{j}$

A) $\mathbf{r}(t) = (-\sin t)\mathbf{i} + (t^4 + 4)\mathbf{j}$
B) $\mathbf{r}(t) = (-\sin t)\mathbf{i} + (t^4)\mathbf{j}$
C) $\mathbf{r}(t) = (-\sin t)\mathbf{i} + (t^4 - 4t + 4)\mathbf{j}$
D) $\mathbf{r}(t) = (\sin t)\mathbf{i} + (12t^2 + 4)\mathbf{j}$

**Solve the problem.**

123) A particle moves along an ellipse in the xy-plane in such a way that its position at time t is $\mathbf{r}(t) = (9\sin 4t)\mathbf{i} + (5\cos 4t)\mathbf{j}$. Find the maximum and minimum values of $|\mathbf{v}|$ and $|\mathbf{a}|$. (*Hint*: Find the extreme values of $|\mathbf{v}|^2$ and $|\mathbf{a}|^2$ first and take square roots later). 123) ______

A) $|\mathbf{v}|_{min} = 5$; $|\mathbf{v}|_{max} = 9$; $|\mathbf{a}|_{min} = 80$; $|\mathbf{a}|_{max} = 144$
B) $|\mathbf{v}|_{min} = 20$; $|\mathbf{v}|_{max} = 36$; $|\mathbf{a}|_{min} = 20$; $|\mathbf{a}|_{max} = 36$
C) $|\mathbf{v}|_{min} = |\mathbf{v}|_{max} = 36$; $|\mathbf{a}|_{min} = |\mathbf{a}|_{max} = 144$
D) $|\mathbf{v}|_{min} = 20$; $|\mathbf{v}|_{max} = 36$; $|\mathbf{a}|_{min} = 80$; $|\mathbf{a}|_{max} = 144$

**Find the unit tangent vector of the given curve.**

124) $\mathbf{r}(t) = \left(9\cos\frac{5}{9}t\right)\mathbf{i} + \left(9\sin\frac{5}{9}t\right)\mathbf{j} - 12t\mathbf{k}$ 124) ______

A) $\mathbf{T}(t) = \left(-\frac{5}{13}\cos 5t\right)\mathbf{i} + \left(\frac{5}{13}\sin 5t\right)\mathbf{j} - \frac{12}{13}\mathbf{k}$

B) $\mathbf{T}(t) = \left(-\frac{5}{13}\sin 5t\right)\mathbf{i} + \left(\frac{5}{13}\cos 5t\right)\mathbf{j} - \frac{12}{13}\mathbf{k}$

C) $\mathbf{T}(t) = \left(-\frac{9}{13}\sin 5t\right)\mathbf{i} + \left(\frac{9}{13}\cos 5t\right)\mathbf{j} - \frac{12}{13}\mathbf{k}$

D) $\mathbf{T}(t) = \left(-\frac{5}{169}\cos 5t\right)\mathbf{i} + \left(\frac{5}{169}\sin 5t\right)\mathbf{j} - \frac{12}{169}\mathbf{k}$

**For the curve r(t), find an equation for the indicated plane at the given value of t.**

125) $\mathbf{r}(t) = (t - 7)\mathbf{i} + (\ln(\sec t) - 5)\mathbf{j} + 8\mathbf{k}$, $-\pi/2 < t < \pi/2$; normal plane at $t = 10\pi$. 125) ______

A) $\mathbf{x} = 10\pi + 7$ B) $\mathbf{x} = 7 - 10\pi$ C) $\mathbf{x} = -10\pi - 7$ D) $\mathbf{x} = 10\pi - 7$

126) $\mathbf{r}(t) = (8t\sin t + 8\cos t)\mathbf{i} + (8t\cos t - 8\sin t)\mathbf{j} + 8\mathbf{k}$; normal plane at $t = 3.5\pi$. 126) ______

A) $y = -8$ B) $x - y + z = -8$ C) $x + y + z = -8$ D) $y = 8$

**Evaluate the integral.**

127) $\int_0^1 \left[(3t^2 - 6)\mathbf{i} + \frac{12t}{t^2+1}\mathbf{j} - \frac{t}{\sqrt{t^2+1}}\mathbf{k}\right] dt$ 127) ______

A) $7\mathbf{i} + 6\ln 2\mathbf{j} + \frac{1-\sqrt{2}}{2}\mathbf{k}$

B) $7\mathbf{i} + 6\ln 2\mathbf{j} + (1 + \sqrt{2})\mathbf{k}$

C) $7\mathbf{i} + 6\ln 2\mathbf{j} + (1 - \sqrt{2})\mathbf{k}$

D) $-5\mathbf{i} + 6\ln 2\mathbf{j} + (1 - \sqrt{2})\mathbf{k}$

**Find the torsion of the space curve.**

128) $\mathbf{r}(t) = (t - 5)\mathbf{i} + (\ln(\cos t) - 4)\mathbf{j} + 7\mathbf{k}$, $-\pi/2 < t < \pi/2$ 128) ______

A) $\tau = -1$ B) $\tau = 0$ C) $\tau = 1$ D) Undefined

**Solve the initial value problem.**

129) Differential Equation: $\frac{d\mathbf{r}}{dt} = \mathbf{i} + (9t^3 - 9t)\mathbf{j} + \frac{1}{t+3}\mathbf{k}$ 129) ______

Initial Condition: $\mathbf{r}(0) = 2\mathbf{j} + (\ln 3)\mathbf{k}$

A) $\mathbf{r}(t) = \mathbf{i} + \left(\frac{t^2}{4}(9t^2 - 18) + 2\right)\mathbf{j} + (\ln 3)\mathbf{k}$

B) $\mathbf{r}(t) = t\mathbf{i} + \left(\frac{t^2}{4}(9t^2 - 18) + 2\right)\mathbf{j} + \ln(t+3)\mathbf{k}$

C) $\mathbf{r}(t) = t\mathbf{i} + \left(\frac{t^2}{2}(9t^2 - 18) + 2\right)\mathbf{j} + (\ln t)\mathbf{k}$

D) $\mathbf{r}(t) = t\mathbf{i} + \left(\frac{t^2}{4}(9t^2 - 18) + 1\right)\mathbf{j} + (\ln 3)\mathbf{k}$

**The vector r(t) is the position vector of a particle in space at time t. Find the time or times in the given time interval when the velocity and acceleration vectors are orthogonal.**

130) $\mathbf{r}(t) = 7\cos t\mathbf{i} + 7\sin t\mathbf{j}$; $0 \le t \le 2\pi$ 130) ______

A) $t = 0, \pi, 2\pi$

B) $t = 0, \pi/2, \pi, 3\pi/2, 2\pi$

C) $0 \le t \le 2\pi$

D) $t = \pi/2, 3\pi/2$

**Calculate the arc length of the indicated portion of the curve r(t).**

131) $\mathbf{r}(t) = (1 + 2t^3)\mathbf{i} + (2t^3 - 5)\mathbf{j} + (3 - t^3)\mathbf{k}$, $-1 \le t \le 2$ 131) ______

A) 27  B) 81  C) 63  D) 21

**The position vector of a particle is r(t). Find the requested vector.**

132) The acceleration at $t = \frac{\pi}{10}$ for $\mathbf{r}(t) = (6 \sin 5t)\mathbf{i} - (5 \cos 5t)\mathbf{j} + (8 \csc 5t)\mathbf{k}$ 132) ______

A) $\mathbf{a}\left(\frac{\pi}{10}\right) = 125\mathbf{j} + 200\mathbf{k}$  B) $\mathbf{a}\left(\frac{\pi}{10}\right) = -150\mathbf{i} - 200\mathbf{k}$

C) $\mathbf{a}\left(\frac{\pi}{10}\right) = 150\mathbf{i} + 200\mathbf{k}$  D) $\mathbf{a}\left(\frac{\pi}{10}\right) = -150\mathbf{i} + 200\mathbf{k}$

**For the curve r(t), write the acceleration in the form $a_T\mathbf{T} + a_N\mathbf{N}$.**

133) $\mathbf{r}(t) = (\cosh t)\mathbf{i} + (\sinh t)\mathbf{j} + t\mathbf{k}$ 133) ______

A) $\mathbf{a} = (-\sinh t)\mathbf{T} + \mathbf{N}$  B) $\mathbf{a} = (-\sqrt{2} \sinh t)\mathbf{T} + \mathbf{N}$

C) $\mathbf{a} = (\sqrt{2} \sinh t)\mathbf{T} + \mathbf{N}$  D) $\mathbf{a} = (\sinh t)\mathbf{T} + \mathbf{N}$

**The position vector of a particle is r(t). Find the requested vector.**

134) The acceleration at t = 1 for $\mathbf{r}(t) = t^5\mathbf{i} + 2\ln\left(\frac{1}{1+t}\right)\mathbf{j} + \frac{1}{t}\mathbf{k}$ 134) ______

A) $\mathbf{a}(1) = 20\mathbf{i} - \frac{1}{2}\mathbf{j} - 2\mathbf{k}$  B) $\mathbf{a}(1) = 20\mathbf{i} + \frac{1}{2}\mathbf{j} - 2\mathbf{k}$

C) $\mathbf{a}(1) = 20\mathbf{i} + 1\mathbf{j} + 2\mathbf{k}$  D) $\mathbf{a}(1) = 20\mathbf{i} + \frac{1}{2}\mathbf{j} + 2\mathbf{k}$

**Solve the problem.**

135) The Tiros II satellite has an orbital eccentricity e =0.001 and a perihelion velocity $v_0$ = 7430 m/s. Find the semimajor axis of the orbit. (G = 6.6720 x $10^{-11}$ $Nm^2kg^{-2}$, mass of earth = 5.975 x $10^{24}$ kg). 135) ______

A) 7.229 x $10^6$ km  B) 7229 km  C) 53,760 km  D) 5.376 x $10^7$ km

136) Find the equation for the circle of curvature (osculating circle) to the space curve $\mathbf{r}(t) = 5t^2\mathbf{i} + 6t\mathbf{j}$ at the point where t = 0. 136) ______

A) $\left(x - \frac{18}{5}\right)^2 + y^2 = \frac{18}{5}$  B) $\left(x + \frac{18}{5}\right)^2 + y^2 = \frac{18}{5}$

C) $\left(x - \frac{5}{18}\right)^2 + y^2 = \frac{5}{18}$  D) $x^2 + y^2 = \frac{18}{5}$

**The vector r(t) is the position vector of a particle in space at time t. Find the time or times in the given time interval when the velocity and acceleration vectors are orthogonal.**

137) $\mathbf{r}(t) = 8 \sin t\mathbf{i} + 4t\mathbf{j}$; $0 \le t \le 2\pi$ 137) ______

A) $t = 0, \pi, 2\pi$  B) $t = \pi/2, 3\pi/2$

C) $0 \le t \le 2\pi$  D) $t = 0, \pi/2, \pi, 3\pi/2, 2\pi$

**Solve the problem. Assume projectile is ideal, launch angle is measured from the horizontal, and launch is over a horizontal surface, unless stated otherwise.**

138) A projectile is fired with an initial speed of 528 m/sec at an angle of 45°. What is the greatest height reached by the projectile? Round answer to the nearest tenth of a meter. 138) ______

A) 28,447.3 m  B) 69,696.0 m  C) 7111.8 m  D) 76.2 m

**For the curve r(t), write the acceleration in the form $a_T\mathbf{T} + a_N\mathbf{N}$.**

139) $\mathbf{r}(t) = (t + 2)\mathbf{i} + (\ln(\cos t) - 3)\mathbf{j} + 8\mathbf{k}$ 139) ______

A) $\mathbf{a} = (\sec t \tan t)\mathbf{T} - (\sec t)\mathbf{N}$
B) $\mathbf{a} = (-\sec t \tan t)\mathbf{T} + (\sec t)\mathbf{N}$
C) $\mathbf{a} = (\sec t \tan t)\mathbf{T} + (\sec t)\mathbf{N}$
D) $\mathbf{a} = (-\sec t \tan t)\mathbf{T} - (\sec t)\mathbf{N}$

140) $\mathbf{r}(t) = \left(2 \sin \frac{5}{2}t + 8\right)\mathbf{i} + \left(2 \cos \frac{5}{2}t - 8\right)\mathbf{j} + 12t\mathbf{k}$ 140) ______

A) $\mathbf{a} = 25\mathbf{N}$
B) $\mathbf{a} = \mathbf{T} + 25\mathbf{N}$
C) $\mathbf{a} = 25\mathbf{T} + 25\mathbf{N}$
D) $\mathbf{a} = 25\mathbf{T}$

**For the curve r(t), find an equation for the indicated plane at the given value of t.**

141) $\mathbf{r}(t) = \frac{40}{9}(1 + t)^{3/2}\mathbf{i} + \frac{40}{9}(1 - t)^{3/2}\mathbf{j} + \frac{10}{3}t\mathbf{k}$; osculating plane at $t = 0$. 141) ______

A) $x - y = -\frac{40}{9}$
B) $x - y = \frac{80}{9}$
C) $x - y = 0$
D) $x - y = \frac{40}{9}$

**Provide an appropriate response.**

142) What two angles of elevation will enable a projectile to reach a target 17 km downrange on the same level as the gun if the projectile's initial speed is 420 m/s? Assume there is no wind resistance. 142) ______

A) 70.82° and 19.18°
B) 35.41°
C) 35.41° and 54.59° [computation result is in degrees. Answer to two decimal places]
D) 0.03° and 89.97°

**For the smooth curve r(t), find the parametric equations for the line that is tangent to r at the given parameter value $t = t_0$.**

143) $\mathbf{r}(t) = (6 \tan t)\mathbf{i} - (3 \sin t)\mathbf{j} + (2 \cos^2 t)\mathbf{k}$ ; $t_0 = \frac{\pi}{4}$ 143) ______

A) $x = 6 + 12t,\ y = -\frac{3}{2} - \frac{3}{2}t,\ z = 2 - 2t$
B) $x = 6 + 12t,\ y = -\frac{3\sqrt{2}}{2} + \frac{3\sqrt{2}}{2}t,\ z = 1 + 2t$
C) $x = 12 + 6t,\ y = -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}t,\ z = -2 + 1t$
D) $x = 6 + 12t,\ y = -\frac{3\sqrt{2}}{2} - \frac{3\sqrt{2}}{2}t,\ z = 1 - 2t$

**The vector r(t) is the position vector of a particle at time t. Find the angle between the velocity and the acceleration vectors at time t = 0.**

144) $\mathbf{r}(t) = \sqrt{2}t\mathbf{i} + (\sqrt{2}t + \frac{\pi}{4}t^2)\mathbf{k}$ 144) ______

A) $\frac{\pi}{4}$
B) $\pi$
C) $\frac{\pi}{2}$
D) 0

**Solve the problem.**

145) At perihelion, Venus' velocity $v_o$ is $3.516 \times 10^4$ m/s. Its orbit has an eccentricity $e = 0.0068$. Find the minimum orbital radius $r_o$ for Venus. ($G = 6.6720 \times 10^{-11}$ $Nm^2kg^{-2}$, mass of sun $= 1.99 \times 10^{30}$ kg). 145) ______

A) $1.081 \times 10^{11}$ km
B) $1.081 \times 10^8$ km
C) $1.147 \times 10^8$ km
D) $1.147 \times 10^{11}$ km

**Calculate the arc length of the indicated portion of the curve r(t).**

146) $\mathbf{r}(t) = (2t \sin t + 2 \cos t)\mathbf{i} + (2 t \cos t - 2 \sin t)\mathbf{j}$ ; $-1 \le t \le 7$ 146) ______

A) 50
B) 100
C) 96
D) 48

**Solve the problem.**

147) The position of a particle is given by $\mathbf{r}(t) = \sin 2t\,\mathbf{i} + \cos 4t\,\mathbf{j}$. Find the velocity vector for the particle. 147) ______

A) $\cos 2t\,\mathbf{i} - \sin 4t\,\mathbf{j}$
B) $2\cos 2t\,\mathbf{i} - 4\sin 4t\,\mathbf{j}$
C) $2\,\mathbf{i} + 4\,\mathbf{j}$
D) $2\sin 2t\,\mathbf{i} + 4\cos 4t\,\mathbf{j}$

**Find T, N, and B for the given space curve.**

148) $\mathbf{r}(t) = (3 + t)\mathbf{i} + (1 + \ln(\sec t))\mathbf{j} - 3\mathbf{k}$, $-\pi/2 < t < \pi/2$ 148) ______

A) $\mathbf{T} = (\cos t)\mathbf{i} - (\sin t)\mathbf{j}$; $\mathbf{N} = (-\sin t)\mathbf{i} - (\cos t)\mathbf{j}$ ; $\mathbf{B} = -\mathbf{k}$
B) $\mathbf{T} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j}$; $\mathbf{N} = (\sin t)\mathbf{i} - (\cos t)\mathbf{j}$ ; $\mathbf{B} = -\mathbf{k}$
C) $\mathbf{T} = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}$; $\mathbf{N} = (-\sin t)\mathbf{i} + (\cos t)\mathbf{j}$ ; $\mathbf{B} = \mathbf{k}$
D) $\mathbf{T} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j}$; $\mathbf{N} = (-\cos t)\mathbf{i} + (\sin t)\mathbf{j}$ ; $\mathbf{B} = \mathbf{k}$

**Solve the problem.**

149) Saturn has a mass of $5.684 \times 10^{26}$ kg and a rotational period of 10.12 hours. What is the radius of a circular geosynchronous orbit around Saturn? ($G = 6.6720 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$). 149) ______

A) $1.262 \times 10^{7}$ km
B) $1.084 \times 10^{8}$ km
C) 12,620 km
D) 108,400 km

**If r(t) is the position vector of a particle in the plane at time t, find the indicated vector.**

150) $\mathbf{r}(t) = (\cos 4t)\mathbf{i} + (5\sin t)\mathbf{j}$ 150) ______
Find the acceleration vector.

A) $\mathbf{a}(t) = (-16\cos 4t)\mathbf{i} + (-5\sin t)\mathbf{j}$
B) $\mathbf{a}(t) = (-16\cos 4t)\mathbf{i} + (-25\sin t)\mathbf{j}$
C) $\mathbf{a}(t) = (16\cos 4t)\mathbf{i} + (-5\sin t)\mathbf{j}$
D) $\mathbf{a}(t) = (-4\cos 4t)\mathbf{i} + (5\sin t)\mathbf{j}$

**For the curve r(t), write the acceleration in the form $a_T\mathbf{T} + a_N\mathbf{N}$.**

151) $\mathbf{r}(t) = (t^2 - 9)\mathbf{i} + (2t - 5)\mathbf{j} + 9\mathbf{k}$ 151) ______

A) $\mathbf{a} = \dfrac{2t}{\sqrt{t^2+1}}\mathbf{T} + \dfrac{2}{\sqrt{t^2+1}}\mathbf{N}$
B) $\mathbf{a} = \dfrac{t}{\sqrt{t^2+1}}\mathbf{T} + \dfrac{1}{\sqrt{t^2+1}}\mathbf{N}$
C) $\mathbf{a} = \dfrac{t}{t^2+1}\mathbf{T} + \dfrac{1}{t^2+1}\mathbf{N}$
D) $\mathbf{a} = \dfrac{2t}{t^2+1}\mathbf{T} + \dfrac{2}{t^2+1}\mathbf{N}$

**Provide an appropriate response.**

152) A baseball is hit when it is 3.2 feet above the ground. It leaves the bat with an initial speed of 150 ft/s, making an angle of 24° with the horizontal. Assuming a drag coefficient k = 0.12, find the range and the flight time of the ball. 152) ______

For projectiles with linear drag:

$$x = x_0 + \frac{v_0}{k}\left(1 - e^{-kt}\right)\cos\alpha,$$

$$y = y_0 + \frac{v_0}{k}\left(1 - e^{-kt}\right)\sin\alpha + \frac{g}{k^2}\left(1 - kt - e^{-kt}\right)$$

where k is the drag coefficient, $v_0$ and $\alpha$ are the projectile's initial speed and launch angle, and g is the acceleration of gravity (32 ft/$s^2$).

A) Flight time = 3.97 s; range = 424.6 feet
B) Flight time = 3.62 s; range = 402.4 feet
C) Flight time = 2.82 s; range = 393.7 feet
D) Flight time = 3.74 s; range = 413.7 feet

**Find the curvature of the space curve.**

153) $\mathbf{r}(t) = (5\ t \sin t + 5 \cos t)\mathbf{i} + 5\mathbf{j} + (5t \cos t - 5 \sin t)\mathbf{k}$ 153) ______

A) $\kappa = \frac{1}{5t}$ B) $\kappa = -\frac{1}{5t}$ C) $\kappa = 5t$ D) $\kappa = \frac{1}{25t^2}$

**Calculate the arc length of the indicated portion of the curve r(t).**

154) $\mathbf{r}(t) = 6\sqrt{2}t^{3/2}\mathbf{i} + (9t \sin t)\mathbf{j} + (9t \cos t)\mathbf{k}$ ; $-4 \le t \le 7$ 154) ______

A) 396 B) 247.5 C) 319.5 D) 612

**Find the length of the indicated portion of the curve.**

155) $\mathbf{r}(t) = (e^t \cos t)\mathbf{i} + (e^t \sin t)\mathbf{j} + 5e^t\mathbf{k}$, $-\ln 2 \le t \le 0$ 155) ______

A) $\frac{3\sqrt{3}}{2}$ B) $\frac{3\sqrt{3}}{4}$ C) $\frac{\sqrt{23}}{2}$ D) $\frac{\sqrt{23}}{4}$

**The position vector of a particle is r(t). Find the requested vector.**

156) The acceleration at t = 0 for $\mathbf{r}(t) = t^2\mathbf{i} + (8t^3 - 8)\mathbf{j} + \sqrt{4 - 2t}\mathbf{k}$ 156) ______

A) $\mathbf{a}(0) = 2\mathbf{i} - \frac{1}{2}\mathbf{k}$ B) $\mathbf{a}(0) = 2\mathbf{i} - \frac{1}{8}\mathbf{k}$ C) $\mathbf{a}(0) = 2\mathbf{i} - \frac{1}{16}\mathbf{k}$ D) $\mathbf{a}(0) = 2\mathbf{i} + \frac{1}{8}\mathbf{k}$

**Solve the problem.**

157) The orbit of the *Syncom 3* satellite has a period of T = 1436.2 minutes. Calculate the semimajor axis of the satellite. (Earth's mass = $5.975 \times 10^{24}$ kg and G = $6.6720 \times 10^{-11}$ $\text{Nm}^2\text{kg}^{-2}$). 157) ______

A) 2751 km B) $2.751 \times 10^6$ km C) 42,170 km D) $4.217 \times 10^7$ km

158) Find the equation for the circle of curvature (osculating circle) to the space curve $\mathbf{r}(t) = 4\sin(t)\mathbf{i} + 4\cos(t)\mathbf{j} + 4\mathbf{k}$ at the point where $t = \frac{\pi}{4}$. 158) ______

A) $\left(x - \frac{\sqrt{2}}{2}\right)^2 + \left(y - \frac{\sqrt{2}}{2}\right)^2 = 1$ B) $x^2 + y^2 = \frac{\sqrt{2}}{2}$

C) $\left(x - \frac{\sqrt{2}}{2}\right)^2 + \left(y - \frac{\sqrt{2}}{2}\right)^2 = \frac{\sqrt{2}}{2}$ D) $\left(x + \frac{\sqrt{2}}{2}\right)^2 + \left(y + \frac{\sqrt{2}}{2}\right)^2 = 1$

159) A satellite is to be placed in a circular equatorial orbit such that it is directly over the same point on earth every 11 hours. At what orbital radius $r_o$ should the satellite be placed? 159) ______

(G = $6.6720 \times 10^{-11}$ $\text{Nm}^2\text{kg}^{-2}$, mass of earth = $5.975 \times 10^{24}$ kg).

A) $r_o$ = 25,111,627.5 km B) $r_o$ = 25,111.6275 km

C) $r_o$ = 19,527,065.9 km D) $r_o$ = 19,527.0659 km

**Solve the problem. Assume projectile is ideal, launch angle is measured from the horizontal, and launch is over a horizontal surface, unless stated otherwise.**

160) A projectile is fired at a speed of 920 m/sec at an angle of 38°. How long will it take to get 23 km downrange? Round answer to the nearest second. 160) ______

A) 34 sec B) 32 sec

C) It will never get that far downrange. D) 30 sec

**Find the torsion of the space curve.**

161) $\mathbf{r}(t) = (\cosh t)\mathbf{i} + t\mathbf{j} + (\sinh t)\mathbf{k}$ 161) ______

A) $\tau = -\dfrac{\text{sech}^2 t}{2}$ B) $\tau = -\dfrac{\tanh^2 t}{2}$ C) $\tau = \dfrac{\tanh^2 t}{2}$ D) $\tau = 0$

1) Given: **v** and **a** are orthogonal. Thus $\mathbf{v}\cdot\mathbf{a} = \mathbf{v}\cdot(d\mathbf{v}/dt) = \frac{1}{2}d(\mathbf{v}\cdot\mathbf{v})/dt = 0$.

So, $\mathbf{v}\cdot\mathbf{v} = |\mathbf{v}|^2$ = constant and thus $|\mathbf{v}|$ = constant.

2) $$\int_a^b (\mathbf{r}_1(t) + \mathbf{r}_2(t))\ dt = \int_a^b \left[(x_1(t)\mathbf{i} + y_1(t)\mathbf{j}) + (x_2(t)\mathbf{i} + y_2(t)\mathbf{j})\right] dt$$

$$= \int_a^b \left[x_1(t)\mathbf{i} + y_1(t)\mathbf{j}\right] dt + \int_a^b \left[x_2(t)\mathbf{i} + y_2(t)\mathbf{j}\right] dt$$

$$= \int_a^b \mathbf{r}_1(t)\ dt + \int_a^b \mathbf{r}_2(t)\ dt$$

3) x–coordinate:

$\frac{d^2x}{dt^2} = 0$, so $\frac{dx}{dt}$ = constant $= v_{o,x} = v_o \cos\alpha$

Then, $x = \int \frac{dx}{dt}\ dt = \int v_o \cos\alpha\ dt = v_o \cos\alpha\ t + x_o$

y–coordinate:

$\frac{d^2y}{dt^2} = -g$ , so $\frac{dy}{dt} = \int \frac{d^2y}{dt^2}\ dt = \int -g\ dt = -gt + v_{y,o} = v_o \sin\alpha - gt$

$y = \int \frac{dy}{dt} dt = \int v_o \sin\alpha - gt\ dt = v_o \sin\alpha\ t - \frac{1}{2}gt^2 + y_o$

4) (a) The dot product of the normals is $\mathbf{n}_1 \cdot \mathbf{n}_2 = |\mathbf{n}_1||\mathbf{n}_2| \cos\theta$,. Solve for θ:

$\theta = \cos^{-1}\left(\frac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}\right)$.

(b) $\theta = 7.87$

5) (a) $\mathbf{T} = \frac{t}{\sqrt{t^2+1}}\mathbf{i} + \frac{1}{\sqrt{t^2+1}}\mathbf{j}$

$\mathbf{N} = \frac{1}{\sqrt{t^2+1}}\mathbf{i} - \frac{t}{\sqrt{t^2+1}}\mathbf{j}$

$\mathbf{B} = -\mathbf{k}$

(b) $\kappa = \frac{1}{2(t^2+1)^{3/2}}$

(c) $\tau = 0$

(d) $\mathbf{a}_T = \frac{2t}{\sqrt{t^2+1}}$; $\mathbf{a}_N = \frac{2}{\sqrt{t^2+1}}$

6) Problem is separable.
x–coordinate:

$$\frac{d^2x}{dt^2} = \frac{dv_x}{dt} = -kv_x \text{ or } \frac{dv_x}{v_x} = -k\,dt. \text{ Integrate to get } \ln\left(\frac{v_x}{v_{x,o}}\right) = -kt \text{ or}$$

$$v_x = v_{x,o}e^{-kt}$$

$$\text{Next, } x = \int v_x\,dt = \int v_{x,o}e^{-kt}\,dt = -\frac{v_{x,o}}{k}e^{-kt} + C$$

$$\text{At } t = 0,\ x(0) = x_o = -\frac{v_{x,o}}{k} + C \quad \text{or} \quad C = x_o + \frac{v_{x,o}}{k}$$

$$\text{Thus, } x = x_o + \frac{v_{x,o}}{k}\left(1 - e^{-kt}\right). \text{ Also, } v_{x,o} = v_o\cos\alpha, \text{ so}$$

$$x = x_o + \frac{v_o}{k}\left(1 - e^{-kt}\right)\cos\alpha.$$

y–coordinate:

$$\frac{d^2y}{dt^2} = \frac{dv_y}{dt} = -g - kv_y. \text{ Let } w = \frac{g + kv_y}{k}, \text{ then } \frac{dw}{dt} = \frac{dv_y}{dt}. \text{ Make substitutions to get:}$$

$$\frac{dw}{dt} = -kw \quad \text{or} \quad \frac{dw}{w} = -k\,dt. \text{ Integrate to get: } \ln\left(\frac{w}{w_o}\right) = -kt \text{ or } w = w_oe^{-kt}.$$

Replace w:

$$\frac{g + kv_y}{k} = \left(\frac{g + kv_{y,o}}{k}\right)e^{-kt}. \text{ Solve for } v_y \text{ to get } v_y = \frac{1}{k}\left(g + kv_{y,o}\right)e^{-kt} - \frac{g}{k}.$$

$$\text{Finally, } y = \int v_y\,dt = \int \frac{1}{k}\left(g + kv_{y,o}\right)e^{-kt} - \frac{g}{k}\,dt = -\frac{1}{k^2}\left(g + kv_{y,o}\right)e^{-kt} - \frac{g}{k}t + C.$$

$$\text{At } t = 0,\ y = y_o = -\frac{1}{k^2}\left(g + kv_{y,o}\right) + C \quad \text{or} \quad C = y_o + \frac{1}{k^2}\left(g + kv_{y,o}\right). \text{ Thus,}$$

$$y = y_o - \frac{1}{k^2}\left(g + kv_{y,o}\right)e^{-kt} - \frac{g}{k}t + \frac{1}{k^2}\left(g + kv_{y,o}\right)$$

$$= y_o + \frac{1}{k^2}\left(g + kv_{y,o}\right)\left(1 - e^{-kt}\right) - \frac{g}{k}t$$

Also, $v_{y,o} = v_o\sin\alpha$. Substituting and rearranging:

$$y = y_o + \frac{v_o}{k}\left(1 - e^{-kt}\right)\sin\alpha + \frac{g}{k^2}\left(1 - kt - e^{-kt}\right)$$

7) Immediately after the wind gust, the velocity is $\mathbf{v} = 121.56\mathbf{i} + 47.88\mathbf{j}$.
Speed: v = 130.65 ft/s.
Flight time: t = 2.99 s
Range: t = 393.71 ft

8) (a) $\mathbf{T} = \cos(t)\mathbf{i} - $ sin(<c>t)$\mathbf{j}$
$\mathbf{N} = -\sin(t)\mathbf{i} - \cos(t)\mathbf{j}$
$\mathbf{B} = -\mathbf{k}$
(b) $\kappa = \cos(t)$
(c) $\tau = 0$
(d) $\mathbf{a}_T = \sec(t)\tan(t)$ ; $\mathbf{a}_N = \sec(t)$

9) (a) Call the length of the perpendicular component d. Since the triangle is a right triangle, $\sin\theta = \dfrac{d}{\left|\overrightarrow{PS}\right|}$ or

$d = \left|\overrightarrow{PS}\right| \sin\theta$.

(b) $\dfrac{\left|\overrightarrow{PS} \times \mathbf{v}\right|}{|\mathbf{v}|} = \dfrac{\left|\overrightarrow{PS}\right| \left|\mathbf{v}\right| \sin\theta}{|\mathbf{v}|} = \left|\overrightarrow{PS}\right| \sin\theta = d$

(c) $d = 4.26$

10) (a) $\mathbf{T} = -\cos(t)\mathbf{i} - \sin(4t)\mathbf{j}$
$\mathbf{N} = \sin(t)\mathbf{i} - \cos(t)\mathbf{j}$
$\mathbf{B} = \mathbf{k}$
(b) $\kappa = \dfrac{1}{3t}$
(c) $\tau = 0$
(d) $\mathbf{a}_T = 3$; $\mathbf{a}_N = 3t$

11) (a) Cylinder: For the curve, $y^2 + z^2 = \cos^2(t) + \sin^2(t) = 1$. Thus, the curve lies on the cylinder $y^2 + z^2 = 1$.

Plane: $t = 0 \Rightarrow P(0, 1, 0)$; $t = \dfrac{\pi}{2} \Rightarrow Q(1, 0, 1)$; $t = \pi \Rightarrow R(2, -1, 0) \Rightarrow \overrightarrow{PQ} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ and $\overrightarrow{PR} = 2\mathbf{i} - 2\mathbf{j} + 0\mathbf{k}$. A vector $\mathbf{n}$ normal to the plane containing P, Q, and R is

$\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = 2\mathbf{i} + 2\mathbf{j} + 0\mathbf{k}$

The equation for this plane that contains the point P is $2x + 2y = 2$ or $x + y = 1$.
For the curve $\mathbf{r}(t)$, $x + y = 1 - \cos(t) + \cos(t) = 1$. So the curve is in the plane $x + y = 1$.
Since the curve $\mathbf{r}(t)$ is the intersection of a cylinder and a plane, it is an ellipse.

(b) $\mathbf{T} = \dfrac{1}{\sqrt{2\sin^2(t) + \cos^2(t)}}\left[\sin(t)\mathbf{i} - \sin(t)\mathbf{j} + \cos(t)\mathbf{k}\right]$

$\mathbf{T}(0) = \mathbf{k}$

$\mathbf{T}\left(\dfrac{\pi}{2}\right) = \dfrac{\sqrt{2}}{2}[\mathbf{i} - \mathbf{j}]$

(c) $\mathbf{a} = \cos(t)\mathbf{i} - \cos(t)\mathbf{j} - \sin(t)\mathbf{k}$. Also, $\mathbf{a} \cdot \mathbf{n} = 2\cos(t) - 2\cos(t) + 0 = 0$. Thus, $\mathbf{a}$ is everywhere perpendicular to the plane's normal and therefore parallel to the plane of the ellipse.

$\mathbf{a}(0) = \mathbf{i} - \mathbf{j}$ and $\mathbf{a}\left(\dfrac{\pi}{2}\right) = -\mathbf{k}$

(d) $L = \displaystyle\int_0^{2\pi} \sqrt{2\sin^2(t) + \cos^2(t)}\, dt$

12) (a) $\mathbf{T} = \tanh(t)\mathbf{i} + \mathbf{j} + \text{sech}(t)\mathbf{k}$

$\mathbf{N} = \text{sech}(t)\mathbf{i} - \tanh(t)\mathbf{k}$

$\mathbf{B} = -\frac{\sqrt{2}}{2}\tanh(t)\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} - \frac{\sqrt{2}}{2}\text{sech}(t)\mathbf{k}$

(b) $\kappa = \frac{\text{sech}^2 t}{2}$

(c) $\tau = \frac{\text{sech}^2 t}{2}$

(d) $\mathbf{a}_T = \sqrt{2}\sinh t$; $\mathbf{a}_N = 1$

13) Flight time to tree: $t = \frac{\text{distance}}{v_o \cos\theta} = 1.789$ seconds;

height at 1.789 seconds: $y = (v_o \sin\theta)t - \frac{1}{2}gt^2 = 36.46$ ft. Tree is 29 ft high

14) (a) $\mathbf{T} = \frac{4}{5}\cos\left(\frac{2}{3}t\right)\mathbf{i} - \frac{4}{5}\sin\left(\frac{2}{3}t\right)\mathbf{j} + \frac{3}{5}\mathbf{k}$

$\mathbf{N} = -\sin\left(\frac{2}{3}t\right)\mathbf{i} - \cos\left(\frac{2}{3}t\right)\mathbf{j} + 0\mathbf{k}$

$\mathbf{B} = \frac{3}{5}\cos\left(\frac{2}{3}t\right)\mathbf{i} - \frac{3}{5}\sin\left(\frac{2}{3}t\right)\mathbf{j} - \frac{4}{5}\mathbf{k}$

(b) $\kappa = \frac{8}{75}$

(c) $\tau = -\frac{12}{25}$

(d) $\mathbf{a}_T = 0$ ; $\mathbf{a}_N = \frac{8}{3}$

15) (a) **T** = cos(t)**i** + sin(<c>t)**j**

$\mathbf{N} = -\sin(t)\mathbf{i} + \cos(t)\mathbf{j}$

$\mathbf{B} = \mathbf{k}$

(b) $\kappa = \cos(t)$

(c) $\tau = 0$

(d) $\mathbf{a}_T = \sec(t)\tan(t)$ ; $\mathbf{a}_N = \sec(t)$

16) $$\frac{d}{dt}(f\,\mathbf{u}) = \frac{d(f(\mathbf{u}_x\mathbf{i} + \mathbf{u}_y\mathbf{j}))}{dt} = \frac{d(f\mathbf{u}_x\mathbf{i} + f\mathbf{u}_y\mathbf{j})}{dt} = \frac{d(f\mathbf{u}_x\mathbf{i})}{dt} + \frac{d(f\mathbf{u}_y\mathbf{j})}{dt}$$

$$= \left(\frac{df}{dt}\mathbf{u}_x + f\frac{d\mathbf{u}_x}{dt}\right)\mathbf{i} + \left(\frac{df}{dt}\mathbf{u}_y + f\frac{d\mathbf{u}_y}{dt}\right)\mathbf{j} = \left(\frac{df}{dt}\mathbf{u}_x\right)\mathbf{i} + \left(\frac{df}{dt}\mathbf{u}_y\right)\mathbf{j} + \left(f\frac{d\mathbf{u}_x}{dt}\right)\mathbf{i} + \left(f\frac{d\mathbf{u}_y}{dt}\right)\mathbf{j}$$

$$= \frac{df}{dt}(\mathbf{u}_x\mathbf{i} + \mathbf{u}_y\mathbf{j}) + f\left(\frac{d\mathbf{u}_x}{dt}\mathbf{i} + \frac{d\mathbf{u}_y}{dt}\mathbf{j}\right) = \frac{df}{dt}\mathbf{u} + f\frac{d\mathbf{u}}{dt}$$

17) Yes: firing angle = 26.1° under these conditions.

18) $$\frac{d}{dt}(\mathbf{u} + \mathbf{v}) = \frac{d}{dt}(u_x\mathbf{i} + u_y\mathbf{j} + v_x\mathbf{i} + v_y\mathbf{j}) = \frac{d}{dt}((u_x + v_x)\mathbf{i} + (u_y + v_y)\mathbf{j})$$

$$= \frac{d}{dt}(u_x + v_x)\mathbf{i} + \frac{d}{dt}(u_y + v_y)\mathbf{j} = \frac{d}{dt}(u_x\mathbf{i} + u_y\mathbf{j}) + \frac{d}{dt}(v_x\mathbf{i} + v_y\mathbf{j}) = \frac{d\mathbf{u}}{dt} + \frac{d\mathbf{v}}{dt}$$

19) $\int_a^b k\mathbf{r}(t)\,dt = \int_a^b k(x(t)\mathbf{i} + y(t)\mathbf{j})\,dt = \int_a^b kx(t)\mathbf{i} + ky(t)\mathbf{j}\,dt$

$$= k\int_a^b x(t)\mathbf{i}\,dt + k\int_a^b y(t)\mathbf{j}\,dt = k\left[\int_a^b x(t)\mathbf{i}\,dt + \int_a^b y(t)\mathbf{j}\,dt\right]$$

$$= k\left[\int_a^b (x(t)\mathbf{i} + y(t)\mathbf{j})\,dt\right] = k\int_a^b \mathbf{r}(t)\,dt$$

20) A
21) A
22) A
23) B
24) D
25) A
26) A
27) A
28) C
29) B
30) C
31) C
32) D
33) B
34) C
35) D
36) C
37) B
38) D
39) C
40) B
41) D
42) D
43) A
44) C
45) B
46) B
47) A
48) D
49) D
50) A
51) B
52) C
53) C
54) D
55) A
56) A
57) A
58) A
59) B
60) C

61) C
62) C
63) C
64) C
65) B
66) C
67) D
68) C
69) D
70) A
71) C
72) D
73) D
74) A
75) A
76) D
77) C
78) C
79) B
80) C
81) A
82) A
83) C
84) D
85) C
86) B
87) A
88) D
89) B
90) B
91) C
92) C
93) A
94) C
95) D
96) A
97) B
98) A
99) D
100) B
101) A
102) D
103) C
104) B
105) B
106) B
107) D
108) D
109) C
110) C

111) A
112) B
113) B
114) D
115) A
116) C
117) A
118) A
119) B
120) B
121) D
122) C
123) D
124) B
125) D
126) A
127) D
128) B
129) B
130) C
131) A
132) D
133) C
134) D
135) B
136) A
137) D
138) C
139) C
140) A
141) C
142) C
143) D
144) A
145) B
146) D
147) B
148) C
149) D
150) A
151) A
152) B
153) A
154) B
155) A
156) B
157) C
158) D
159) D
160) B

161) A