1. Show that the line integral

$$\mathcal{I} = \int_{\mathcal{C}} \left(2x\cos z - x^2\right) dx + (z - 2y)\, dy + \left(y - x^2 \sin z\right) dz$$

is independent of path and evaluate $\mathcal{I}$, where $\mathcal{C}$ runs from $(3, -2, 0)$ to $(1, 0, \pi)$.

2. Evaluate

$$\int_{\mathcal{C}} (xy + y + z)\, d\mathbf{s}$$

where $\mathcal{C}$ is the line segment from $(0, 0, 2)$ to $(2, 1, 0)$.

3. Evaluate the line integral

$$\mathcal{I} = \oint_{\mathcal{C}} -xy^2\,dx + xy^2\ dy$$

where $\mathcal{C}$ is the boundary of the semicircular disk bounded by the x-axis and $y = \sqrt{4 - x^2}$.

4. Find the flux $\left(\iint_S \mathbf{F} \bullet \mathbf{n}\, d\mathbf{S}\right)$ of the field

$$\mathbf{F} = xze^y\mathbf{i} - xze^y\mathbf{j} + z\mathbf{k}$$

across the surface S where S is the part of the plane $x + y + z = 1$ in the first octant with outward orientation.

5. Find the surface area of the part of $z = y^2 - x^2$ that lies between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

6. Use the Divergence Theorem to compute

$$\iint_S \mathbf{F} \bullet \mathbf{n} \; d\mathcal{S}$$

where

$$\mathbf{F} = \left(x^3 + xy^2 + xz^2\right)\mathbf{i} + \left(x^2y + y^3 + yz^2\right)\mathbf{j} + \left(zx^2 + zy^2 + z^3\right)\mathbf{k}$$

and S is bounded by $x^2 + y^2 = 4,\ z = 0$ and $z = 3.$

7. ($\mathfrak{Bonus}$) Let

$$\mathbf{F} = (\sin y + (x+1)^x)\,\mathbf{i} + \left(x\cos y + x^2\right)\mathbf{j}$$

(a) Show that

$$\oint_{\mathcal{C}_a} \mathbf{F}\bullet\, d\mathbf{r} = 0$$

for every a, where $\mathcal{C}_a$ is the circle of radius a centered at the origin.

(b) Is $\oint_{\mathcal{C}} \mathbf{F}\bullet\, d\mathbf{r} = 0$ path independent for any closed piecewisely smooth curve $\mathcal{C}$? Explain.