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DEPARTMENT OF MATHEMATICS
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Math 156
Calculus II
Worksheet 5

Problems

1. In Exercises 1 - 6, determine whether the given vector field is conservative, and find a potential if it is.

$$\mathbf{F}(x,y,z) = x\mathbf{i} - 2y\mathbf{j} + 3z\mathbf{k}$$

2.

$$\mathbf{F}(x,y,z) = y\mathbf{i} + x\mathbf{j} + z^2\mathbf{k}$$

3.

$$\mathbf{F}(x,y) = \frac{x\mathbf{i}+y\mathbf{j}}{x^2+y^2}$$

4.

$$\mathbf{F}(x,y) = \frac{x\mathbf{i}+y\mathbf{j}}{x^2+y^2}$$

5.

$$\mathbf{F}(x,y,z) = \left(2xy - z^2\right)\mathbf{i} + \left(2yz + x^2\right)\mathbf{j} - \left(2zx - y^2\right)\mathbf{k}$$

6.

$$\mathbf{F}(x,y,z) = e^{x^2+y^2+z^2}\left(xz\mathbf{i} + yz\mathbf{j} + xy\mathbf{k}\right)$$

7. Show that the vector field

$$\mathbf{F}(x,y,z) = \frac{2x}{z}\mathbf{i} + \frac{2y}{z}\mathbf{j} - \frac{x^2+y^2}{z^2}\mathbf{k}$$

8. Repeat Exercise 7 for the field

$$\mathbf{F}(x,y,z)=\frac{2x}{z}\mathbf{i}+\frac{2y}{z}\mathbf{j}+\left(1-\frac{x^2+y^2}{z^2}\right)\mathbf{k}$$

9. Let $\mathcal{C}$ be the conical helix with parametric equations

$$x=t\cos t,\ y=t\sin t,\ z=t,\ (0\le t\le 2\pi)\,..\text{Find } \int_{\mathcal{C}} z\ d\mathbf{s}.$$

10. Evaluate

$$\int_{\mathcal{C}} e^z\ d\mathbf{s}$$

where $\mathcal{C}$ is the curve that has parametric representation $x=\cos t,\ y=\sin t,\ z=\cos^2 t,\quad (0\le t\le 2\pi)$..

11. Find

$$\int_{\mathcal{C}} x^2\ d\mathbf{s}$$

along the line of intersection of the two planes $x-y+z=0$, and $x+y+2z=0$, from the origin to the point $(3,1,-2)$

12. Find the work done by the force field

$$\mathbf{F}(x,y,z)=(x+y)\,\mathbf{i}+(x-z)\,\mathbf{j}-(z-y)\,\mathbf{k}$$

in moving an object from $(1,0,-1)$ to $(0,-2,3)$ along any smooth curve.

13. Evaluate

$$\oint x^2y^2\ dx+x^3y\ dy$$

counterclockwise around the square with vertices $(0,0)\,,(1,0)\,,(1,1)\,,$ and $(0,1)\,.$

14. Evaluate

$$\int_{\mathcal{C}} e^{x+y}\sin(y+z)\ dx+e^{x+y}\left(\sin(y+z)+\cos(y+z)\right)\ dy+e^{x+y}\cos(y+z)\ dz$$

along the straight line segment form $(0,0,0)$ to $\left(1,\frac{\pi}{4},\frac{\pi}{4}\right)$..

15. Determine the values of A and B for which the vector field

$$\mathbf{F}(x,y,z) = Ax\ln z\mathbf{i} + By^2z\mathbf{j} + \left(\frac{x^2}{z} + y^3\right)\mathbf{k}$$

is conservative. If $\mathcal{C}$ is the straight line from $(1,1,1)$ to $(2,1,2)$, find

$$\int_{\mathcal{C}} 2x\ln z\ dx + 2y^2z\ dy + y^3\ dz.$$

16. If $\mathcal{C}$ is the intersection of $z = \ln(1+x)$ and $y = x$ from $(0,0,0)$ to $(1,1,\ln 2)$ evaluate

$$\int_{\mathcal{C}} (2x\sin(\pi y) - e^x)\ dx + \left(\pi x^2\cos(\pi y) - 3e^z\right)\ dy - xe^z\ dz.$$

17. Evaluate

$$\oint_{\mathcal{C}} \left(\sin x + 3y^2\right)\ dx + \left(2x - e^{-y^2}\right)\ dy,$$

where $\mathcal{C}$ is the boundary of the half disk $x^2 + y^2 \leq a^2,\ y \geq 0,$ oriented counterclockwise.

18. Evaluate

$$\oint_{\mathcal{C}} \left(x^2 - xy\right)\ dx + \left(xy - y^2\right)\ dy,$$

clockwise around the triangle with vertices $(0,0)$, $(1,1)$,and $(2,0)$..

19. Evaluate

$$\oint_{\mathcal{C}} \left(x\sin\left(y^2\right) - y^2\right)\ dx + \left(x^2y\cos\left(y^2\right) + 3x\right)\ dy,$$

where $\mathcal{C}$ the counterclockwise boundary of the trapezoid with vertices $(0,-2)$, $(1,-1)$, $(1,1)$ and $(0,2)$.

20. Evaluate

$$\oint_{\mathcal{C}} x^2y\ dx - xy^2\ dy,$$

where $\mathcal{C}$ is the clockwise boundary of the region $0 \leq y \leq \sqrt{9 - x^2}$.

21. Find the flux of $\mathbf{F} = x\mathbf{i} + \mathbf{zj}$ out of the tetrahedron bounded by the coordinate planes and the plane $x + 2y + 3z = 6$.

22. Find the flux of $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ outward across the sphere $x^2 + y^2 + z^2 = a^2$.

23. Find the flux of the vector field $\mathbf{F} = y\mathbf{i} + z\mathbf{k}$ out across the boundary of the solid cone $0 \le z \le 1 - \sqrt{x^2 + y^2}$

24. Find the flux of $\mathbf{F} = x\mathbf{i} + x\mathbf{j} + \mathbf{k}$ upward through the part of the surface $z = x^2 - y^2$ lying inside the cylinder $x^2 + y^2 = a^2$.

25. Find the flux of $\mathbf{F} = y^3\mathbf{i} + z^2\mathbf{j} + x\mathbf{k}$ downward through the part of the surface $z = 4 - x^2 - y^2$ that lies above the plane $z = 2x + 1$.

26. Find

$$\iint_S y \, d\mathbf{S}$$

where $\mathcal{S}$ is the part of the plane $z = 1 + y$ that lies inside the cone $z = \sqrt{2\left(x^2 + y^2\right)}$.

27. Find $\displaystyle\iint_S y \, d\mathbf{S}$,where $\mathcal{S}$ is the part of the cone $z = \sqrt{2\left(x^2 + y^2\right)}$ that lies below the plane $z = 1 + y$.

28. Find $\displaystyle\iint_S xz \, d\mathbf{S}$,where S is the part of the surface $z = x^2$ that lies in the first octant of the $3-$space and inside the paraboloid $z = 1 - 3x^2 - y^2$.