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DEPARTMENT OF MATHEMATICS
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Math 156
Calculus II
Worksheet for Midterm 1

Problems

1. Decide where the series

$$a. \sum_{n=0}^{\infty} \frac{1}{\sqrt{n}}(2x-3)^n, \qquad b. \sum_{n=0}^{\infty} \frac{(-1)^n}{n2^n}x^n \tag{1}$$

$$c. \sum_{n=0}^{\infty} \frac{n^2}{10^n}x^n, \qquad d. \sum_{n=0}^{\infty} \frac{n}{4^n}(2x-1)^n, \tag{2}$$

$$e. \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)!}x^{2n-1} \quad f. \sum_{n=1}^{\infty} (-1)^n \frac{(x-1)^n}{\sqrt{n}} \qquad g. \sum_{n=1}^{\infty} \frac{(x-4)^n}{n5^n} \tag{3}$$

is centered at and then determine its radius of convergence and its complete interval of convergence.

2. Determine the exact sum of the infinite series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} e^2}{e^{2n+2}}x^2$$

3. If k is a positive integer, find the radius of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(n!)^k}{(kn!)}x^n$$

4. Verfy that the given functions satisfy the given differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0, where u = \left(x^2+y^2+z^2\right)^{-1/2} \tag{4}$$

$$x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z} = -2w, where w = \left(x^2+y^2+z^2\right)^{-1} \tag{5}$$

5. Find the points on the surface $x^2+y^2+z^2=9$ where the tangent plane is parallel to the plane where $2x-2y+z=0$

6. Let

$$f(x,y)=\begin{cases}\frac{x^3}{x^2+y^2} & if \quad (x,y)\neq(0,0)\\ 0 & if \quad (x,y)=(0,0)\end{cases}$$

.Calculate each of the following partial derivatives or explain why it does not exist: $f_x(0,0), f_y(0,0)\, f_{xy}(0,0), f_{yx}(0,0)$.

7. Let

$$f(x,y)=\frac{x^3-y^3}{x^2-y^2}$$

.Where is $f(x,y)$ is continuous?

8. Find the Maclaurin series representationsfor the functions given below. For what values of x is each representation valid?

$$e^{3x+1},\ \cos\left(2x^3\right),\ \sin(x-\pi/4),\ \cos(2x-\pi),\ x^2\sin(x/3),\ \cos^2(x/2) \quad (6)$$

$$\sin x\cos x,\ \tan^{-1}\left(5x^2\right),\ \frac{1+x^3}{1+x^2},\ \ln\left(2+x^2\right),\ \ln\frac{1-x}{1+x},\ \left(e^{2x^2}-1\right)/x^2 \quad (7)$$

$$\cosh x-\cos x,\ \sinh x-\sin x \quad (8)$$

9. Find the required Taylor series representation of the functions given below. Where is each representation valid?

$$f(x) = e^{-2x} about the point x=-1 \quad (9)$$

$$f(x) = \ln x in powers of x-3 \quad (10)$$

$$f(x) = xe^x in powers of x+2 \quad (11)$$

$$f(x) = \frac{x}{x+1} in powers of x-1 \quad (12)$$

10. Find the sums of the series below

$$1+x^2+\frac{x^4}{2!}+\frac{x^6}{3!}+\frac{x^8}{4!}+\ \cdots \quad (13)$$

$$x^3-\frac{x^9}{3!\times4}+\frac{x^{15}}{5!\times16}-\frac{x^{21}}{7!\times64}+\frac{x^{27}}{9!\times256}-\cdots \quad (14)$$

$$1+\frac{x^2}{3!}+\frac{x^4}{5!}+\frac{x^6}{7!}+\frac{x^8}{9!}+\cdots \quad (15)$$

$$1+\frac{1}{2\times2!}+\frac{1}{4\times3!}+\frac{1}{8\times4!}+\cdots \quad (16)$$

11. Evaluate the limits

(a) $\lim_{x\to 0} \frac{\sin(x^2)}{\sinh x}$,(b) $\lim_{x\to 0} \frac{1-\cos(x^2)}{(1-\cos x)^2}$, (c) $\lim_{x\to 0} \frac{(e^x-1-x)^2}{x^2-\ln(1+x^2)}$,
(d) $\lim_{x\to 0} \frac{2\sin(3x)-3\sin 2x}{5x-\tan^{-1}5x}$

12. Use Maclaurin or Taylor series to calculate the function values below with error less than 5×10^{-5} in absolute value

(a) $e^{1.2}$,(b) $\ln(0.9)$, (c) $\cos 65^0$

13. Find all the first partial derivatives of the function specified and evaluate them at the given point

(a) $f(x,y) = \sin(x\sqrt{y})$ at $\left(\frac{\pi}{3},4\right)$ (b) $f(x,y) = \frac{1}{\sqrt{x^2+y^2}}$ at $(-3,4)$, (c) $w = x^{(y\ln z)}$ at $(e,2e)$, (d) $g(x_1,x_2,x_3,x_4) = \frac{x_1-x_2^2}{x_3+x_4^2}$at $(3,1,-1,-2)$.

14. If $z=\sin(x^2y)$, where $x=st^2$ and $y=s^2+\frac{1}{t}$, find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$

(a) by direct substitution, (b) by using the (two-variable) chain rule.

15. Find the derivatives

$$\frac{\partial}{\partial x}f\left(x^2y,x+2y\right) and \frac{\partial}{\partial y}f\left(x^2y,x+2y\right)$$

in terms of the partial derivatives of f, assuming these partial derivatives are continuous.

16. Find each of the directional derivatives

(a)

$$D_{\mathbf{u}}f(2,0) \textbf{ where } f(x,y) = x\cos y \quad (17)$$
$$inthedirectionuof\theta = \frac{2\pi}{3}. \quad (18)$$

(b)

$$D_{\mathbf{u}}f(x,y,z) \textbf{ where } f(x,y,z) = \sin yz + \ln x^2 \quad (19)$$
$$inthedirectionofu = i+j-k. \quad (20)$$