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DEPARTMENT OF MATHEMATICS
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Math 156
Calculus II
Worksheet 2

Problems

1. Find and classify the critical points of the given functions.

$$\begin{aligned}
(a).f(x,y) &= x^2+2y^2-4x+4y, (b).f(x,y)=xy-x+y \\
(c).f(x,y) &= x^3+y^3-3xy,\ (d)\ f(x,y)=x^4+y^4-4xy \\
(e).f(x,y) &= \frac{x}{y}+\frac{8}{x}-y \quad (f).f(x,y)=\cos(x+y) \\
(g).f(x,y) &= x\sin y \quad (h).\ f(x,y)=\cos x+\cos y, \\
(i)\ f(x,y) &= x^2ye^{-\left(x^2+y^2\right)} \quad (j)\ .f(x,y)=\frac{xy}{2+x^4+y^4} \\
(k)\ f(x,y) &= xe^{-x^3+y^3} \quad (l)\ f(x,y)=\frac{1}{1-x+y+x^2+y^2} \\
(m).f(x,y) &= \left(1+\frac{1}{x}\right)\left(1+\frac{1}{y}\right)\left(\frac{1}{x}+\frac{1}{y}\right) \\
(n)\ f(x,y) &= xyz-x^2-y^2-z^2
\end{aligned}$$

2. Find the maximum and minimum values of

$$f(x,y,z)=xye^{-x^2-y^4}$$

3. Find the maximum and minimum values of

$$f(x,y)=\frac{x}{1+x^2+y^2}$$

4. Find the maximum and minimum values of

$$\begin{aligned}
f(x,y) &= x-x^2+y^2 \text{ on the rectangle} \\
0 &\leq x\leq 2,\ 0\leq y\leq 1.
\end{aligned}$$

5. Find the maximum and minimum values of

$$\begin{aligned}
f(x,y) &= xy-2x \text{ on the rectangle} \\
-1 &\leq x\leq 1,\ 0\leq y\leq 1.
\end{aligned}$$

6. Find the maximum and the minimum values

$$\begin{aligned}
f(x,y) &= xy-y^2 \text{ on the disk} \\
x^2+y^2 &\leq 1.
\end{aligned}$$

7. Find the maximum and minimum values of

$$\begin{aligned} f(x,y) &= xy - x^3y^2 \text{ over the square} \\ 0 &\leq x \leq 1,\, 0 \leq y \leq 1. \end{aligned}$$

8. Find the maximum and minimum values of

$$\begin{aligned} f(x,y) &= xy(1-x-y) \text{ over the triangle with vertices} \\ & (0,0),(1,0),\ \text{and}\ (0,1) \end{aligned}$$

9. Maximize

$$x^3y^5$$

subject to the constraint $x + y = 8$.

9. Find the maximum and minimum values of

$$f(x,y,z) = x + y - z$$

over the sphere $x^2 + y^2 + z^2 = 1$.
10. Maximize

here is $f(x,y)$ is continuous?
9. Find the Maclaurin series representationsfor the functions given below. For what values of x is each representation valid?

$$\begin{aligned} & e^{3x+1},\ \cos\left(2x^3\right),\ \sin(x-\pi/4),\ \cos(2x-\pi),\ x^2\sin(x/3),\ \cos^2(x/2) \\ & \sin x\cos x,\ \tan^{-1}\left(5x^2\right),\ \frac{1+x^3}{1+x^2},\ \ln\left(2+x^2\right),\ \ln\frac{1-x}{1+x},\ \left(e^{2x^2}-1\right)/x^2 \\ & \cosh x - \cos x,\ \sinh x - \sin x \end{aligned}$$

10. Find the required Taylor series representation of the functions given below. Where is each representation valid?

$$\begin{aligned} f(x) &= e^{-2x} \text{ about the point } x = -1 \\ f(x) &= \ln x \text{ in powers of } x - 3 \\ f(x) &= xe^x \text{ in powers of } x + 2 \\ f(x) &= \frac{x}{x+1} \text{ in powers of } x - 1 \end{aligned}$$

11. Find the sums of the series below

$$1+x^2+\frac{x^4}{2!}+\frac{x^6}{3!}+\frac{x^8}{4!}+\ \cdots$$

$$x^3-\frac{x^9}{3!\times 4}+\frac{x^{15}}{5!\times 16}-\frac{x^{21}}{7!\times 64}+\frac{x^{27}}{9!\times 256}-\cdots$$

$$1+\frac{x^2}{3!}+\frac{x^4}{5!}+\frac{x^6}{7!}+\frac{x^8}{9!}+\cdots$$

$$1+\frac{1}{2\times 2!}+\frac{1}{4\times 3!}+\frac{1}{8\times 4!}+\cdots$$

12. Evaluate the limits

$$\text{(a) } \lim_{x\to 0}\frac{\sin\left(x^2\right)}{\sinh x},\text{(b) } \lim_{x\to 0}\frac{1-\cos\left(x^2\right)}{\left(1-\cos x\right)^2},\ \text{(c) } \lim_{x\to 0}\frac{\left(e^x-1-x\right)^2}{x^2-\ln\left(1+x^2\right)},$$
$$\text{(d)} \lim_{x\to 0}\frac{2\sin\left(3x\right)-3\sin 2x}{5x-\tan^{-1}5x}$$

13. Use Maclaurin or Taylor series to calculate the function values below with error less than 5×10^{-5} in absolute value

(a) $e^{1.2}$,(b) $\ln\left(0.9\right)$, (c) $\cos 65^0$

14. Find all the first partial derivatives of the function specified and evaluate them at the given point

(a) $f\left(x,y\right) = \sin\left(x\sqrt{y}\right)$ at $\left(\frac{\pi}{3},4\right)$ (b) $f\left(x,y\right) = \frac{1}{\sqrt{x^2+y^2}}$ at $\left(-3,4\right)$, (c) $w = x^{\left(y\ln z\right)}$ at $(e,2e)$, (d) $g\left(x_1,x_2,x_3,x_4\right) = \frac{x_1-x_2^2}{x_3+x_4^2}$ at $(3,1,-1,-2)$.

15. If $z=\sin\left(x^2y\right)$, where $x=st^2$ and $y=s^2+\frac{1}{t}$, find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$

(a) by direct substitution, (b) by using the (two-variable) chain rule.

16. Find
$$\frac{\partial}{\partial x}f\left(x^2y,x+2y\right)\ \text{and}\ \frac{\partial}{\partial y}f\left(x^2y,x+2y\right)$$
in terms of the partial derivatives of f, assuming these partial derivatives are continuous.