1. (20 pts.) Find a power series solution of the form

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

for the differential equation

$$\left(1 - x^2\right) y'' - 2xy' + 2y = 0.$$

2. Find the Laplace transform of the given functions.

(a) (10 pts.)

$$f(t) = 5 - t^2 + 10t^5 + t^{\frac{5}{2}}$$

(b) (10 pts.)

$$g(t) = 3\cos(5t) - 9\cosh(5t) + 13e^{-7t}\cos(5t)$$

3. (20 pts.) Find the inverse Laplace transform

$$G(s) = \frac{11s + 50}{(2s + 5)(s - 2)}.$$

4. (25 pts.) Use Laplace transforms to solve the given IVP.

$$\begin{aligned} 2y'' + y' - 21y &= 4 - 10u_6(t) - 2u_{13}(t), \\ y(0) &= 0, \quad y'(0) = 8. \\ \text{where } u_c(t) &= \begin{cases} 0 & \text{if} \quad t < c \\ 1 & \text{if} \quad t > c \end{cases} \end{aligned}$$

5. (20 pts.) Find the Fourier series of the function

$$f(x) = \begin{cases} 1 & \text{if} \quad -\pi/2 < x < \pi/2 \\ -1 & \text{if} \quad \pi/2 < x < 3\pi/2 \end{cases}$$

which is assumed to have period 2π.