

1. (20 pts.) Solve the given IVP

$$\frac{1}{x^2} \frac{dy}{dx} = \frac{e^{x+x^3}}{x} + 3y, \quad y(1) = -5.$$

2. (20 pts.) Find the Fourier series for

$$f(x) = |\sin x| \text{ on } -\pi < x < \pi.$$

3. (20 pts.) Find the Fourier series solution of the wave problem

$$\begin{aligned}u_{tt} &= u_{xx}, \quad 0 < x < 1 \\u(0, t) &= 0, \quad u(1, t) = 0 \\u(x, 0) &= \begin{cases} x & \text{if } 0 \leq x \leq 1/2 \\ 1 - x & \text{if } 1/2 \leq x \leq 1 \end{cases} \\u_t(x, 0) &= 1, \quad 0 \leq x \leq 1.\end{aligned}$$

4. (25 pts.) Use Laplace transforms to solve the given IVP.

$$\begin{aligned}y'' + 2y' + 5y &= r(t) \\ y(0) &= 0, \quad y'(0) = 0. \\ \text{where } r(t) &= \begin{cases} 5 & \text{if } 0 < t < \pi \\ 1 & \text{if } \pi < t \end{cases}.\end{aligned}$$

5. (20 pts.) Solve the heat equation

$$u_t(x, t) = u_{xx}(x, t), \quad 0 < x < \pi, t > 0$$

$$u(0, t) = 0, \quad u_x(\pi, t) = 0$$

$$u(x, 0) = x.$$