

1. (20 pts.) Solve the given IVP

$$\frac{dy}{dx} = \frac{y}{x} - \frac{1}{2} \frac{y^3}{x^3}, \quad y(1) = 1.$$

2. (20 pts.) Find the Fourier series for

$$f(x) = \begin{cases} 0 & \text{if } -\pi < x < 0 \\ 1 & \text{if } 0 < x < \pi \end{cases}.$$

3. (20 pts.) Solve the one-dimensional wave equation

$$u_{tt} = 9u_{xx}$$

for  $0 \leq x \leq 1$  with boundary conditions  $u(0, t) = 0$  and  $u(1, t) = 0$  and with initial conditions

$$u(x, 0) = \sin 2\pi x + 5 \sin 3\pi x \quad \text{and} \quad u_t(x, 0) = 3 \sin 5\pi x.$$

4. (25 pts.) Use Laplace transforms to solve the given IVP.

$$y'' - 4y' = e^{2t} + 10u_3(t) e^{2t-6}$$

$$y(0) = 0, \quad y'(0) = 0.$$

$$\text{where } u_c(t) = \begin{cases} 0 & \text{if } t < c \\ 1 & \text{if } t > c \end{cases}.$$

5. (20 pts.) Solve the heat equation

$$u_t = u_{xx}$$

for  $0 \leq x \leq L$  with boundary conditions  $u(0, t) = 0$  and  $u(L, t) = 50$  and with initial condition  $u(x, 0) = 0$ .