ÇANKAYA UNIVERSITY
DEPARTMENT OF MATHEMATICS
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Math 156
Calculus II
Worksheet 3

Problems

In Exercises 1 - 4, calculate the given iterated integral

1.
$$\int_0^1 dx \int_0^x \left(xy + y^2\right)\, dy$$

2.
$$\int_0^1 \int_0^y \left(xy + y^2\right) dx\, dy$$

3.
$$\int_0^\pi \int_{-x}^x \cos y dy dx$$

4.
$$\int_0^2 dy \int_0^y y^2 e^{xy}\, dx$$

In Exercises 5 - 14, evaluate the double integrals by iteration.

5.
$$\iint_R \left(x^2 + y^2\right) dA$$

where R is the rectangle $0 \le x \le a, 0 \le y \le b$

6.
$$\iint_R \left(x^2 y^2\right) dA$$

where R is the rectangle of Exercise 5.

7.
$$\iint_S (\sin x + \cos y)\, dA$$

where S is the square $0 \le x \le \pi/2, 0 \le y \le \pi/2$.

8.
$$\iint_T (x - 3y)\, dA$$

where T is the triangle with vertices $(0,0)\,, (a,0)\,,$ and $(0,b)$

9.
$$\iint_R xy^2 dA$$

where R is the finite region in the first quadrant bounded by the curves $y = x^2$ and $x = y^2$

10.

$$\iint_D x \cos y dA$$

where D is the finite region in the first quadrant bounded by the coordinate axes and the curve $y = 1 - x^2$.

11.

$$\iint_D \ln x dA$$

where D is the finite region in the first quadrant bounded by the line $2x + 2y = 5$ and the hyperbola $xy = 1$.

12.

$$\iint_T \sqrt{a^2 - y^2} dA$$

where T is the triangle with vertices $(0,0), (a,0)$ and (a,a).

13.

$$\iint_R \frac{x}{y^2} e^y dA$$

where R is the region $0 \leq x \leq 1, x^2 \leq y \leq x$.

14.

$$\iint_T \frac{xy}{1+x^4} dA$$

where T is the triangle with vertices $(0,0), (1,0)$ and $(1,1)$.

In Exercises 15 - 18, sketch the domain of integration and evaluate the given iterated integrals

15.

$$\int_0^1 dy \int_y^1 e^{-x^2}\, dx$$

16.

$$\int_0^{\pi/2} dy \int_y^{\pi/2} \frac{\sin x}{x}\, dx$$

17.

$$\int_0^1 dx \int_x^1 \frac{y^\lambda}{x^2+y^2}\, dy$$

18.

$$\int_0^1 dx \int_x^{x^{1/3}} \sqrt{1-y^4}\, dy$$

In Exercises 19 - 28, find the volumes of the indicated solids.

19. Under $z = 1 - x^2$ and above the region $R : 0 \leq x \leq 1,\ 0 \leq y \leq x$

20. Under $z = 1 - x^2$ and above the region $R : 0 \leq y \leq 1,\ 0 \leq x \leq y$

21. Under $z = 1 - x^2 - y^2$ and above the region $x \geq 0,\ y \geq 0,\ x + y \leq 1$

22. Under $z = 1 - y^2$ and above $z = x^2$

23. Under the surface $z = 1/(x + y)$ and above the region in the $xy-$plane bounded by $x = 1, x = 2, , y = 0$,and $y = x$.

24. Under the surface $z = x^2 \sin(y^4)$ and above the triangle in the $xy-$plane with vertices $(0,0), (0, \pi^{1/4})$ and $(\pi^{1/4}, \pi^{1/4})$.

25. Above the $xy-$plane and under the surface $z = 1 - x^2 - 2y^2$

26. Above the triangle with vertices $(0,0), (a,0), (0,b)$ and under the plane $z = 2 - (x/a) - (y/b)$.

27. Inside the two cylinders $x^2 + y^2 = a^2$ and $y^2 + z^2 = a^2$.

28. Inside the cylinder $x^2 + 2y^2 = 8$, above the plane $z = y - 4$ and below the plane $z = 8 - x$

29. Evaluate the integrals

a) Evaluate

$$\int_0^1 \int_0^1 \int_0^1 \left(x^2 + y^2 + z^2\right)\ dz\ dy\ dx$$

b)

$$\int_0^{\sqrt{2}} \int_0^{3y} \int_{x^2+3y^2}^{8-x^2-y^2} dz\ dx\ dy$$

c)

$$\int_0^e \int_0^e \int_0^e \frac{1}{xyz}\ dx\ dy\ dz$$

d)

$$\int_0^1 \int_0^{3-3x} \int_0^{3-3x-y} dz\ dy\ dx$$

e)

$$\int_0^1 \int_0^{\pi} \int_0^{\pi} y \sin z\ dx\ dy\ dz$$

f)

$$\int_{-1}^1 \int_{-1}^1 \int_{-1}^1 (x + y + z)\ dy\ dx\ dz$$

g)

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_0^{2x+y} dz\ dx\ dy$$

h)
$$\int_0^1 \int_0^{2-x} \int_0^{2-x-y} dz \ dy \ dx$$

ı)
$$\int_0^1 \int_0^{1-x^2} \int_0^{4-x^2-y} x \ dz \ dy \ dx$$

j)
$$\int_0^{\pi} \int_0^{\pi} \int_0^{\pi} \cos(u+v+w) \ du \ dv \ dw$$

k)
$$\int_1^e \int_1^e \int_1^e \ln r \ln s \ln t \ dt \ dr \ ds$$

l)
$$\int_0^{\pi/4} \int_0^{\ln \sec v} \int_{-\infty}^{2t} e^x \ dx \ dt \ dv$$

30) Evaluate the integrals by changing the order of integration in an appropriate way.

a)
$$\int_0^4 \int_0^1 \int_{2y}^2 \frac{4\cos(x^2)}{2\sqrt{x}} \ dx \ dy \ dz$$

b)
$$\int_0^1 \int_0^1 \int_{x^2}^1 12xze^{zy^2} \ dy \ dx \ dz$$

c
$$\int_0^1 \int_{\sqrt[3]{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin(\pi y^2)}{y^2} \ dx \ dy \ dz$$

d)
$$\int_0^2 \int_0^{4-x^2} \int_0^x \frac{\sin 2z}{4-z} \ dy \ dz \ dx$$