ÇANKAYA UNIVERSITY
DEPARTMENT OF MATHEMATICS
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Math 156
Calculus II
Worksheet 4

Problems

Evaluate the cylindrical coordinate integrals in Exercises 1 - 6.

1.
$$\int_0^{2\pi}\int_0^{1}\int_r^{\sqrt{2-r^2}} dz\ r\ \ dr\ d\theta$$

2.
$$\int_0^{2\pi}\int_0^{3}\int_{r^2/3}^{\sqrt{18-r^2}} dz\ r\ \ dr\ d\theta$$

3.
$$\int_0^{2\pi}\int_0^{1}\int_r^{\sqrt{2-r^2}} dz\ r\ \ dr\ d\theta$$

4.
$$\int_0^{\pi}\int_0^{\theta/\pi}\int_{-\sqrt{4-r^2}}^{3\sqrt{4-r^2}} z\ dz\ r\ \ dr\ d\theta$$

5.
$$\int_0^{2\pi}\int_0^{1}\int_0^{1/\sqrt{2-r^2}} 3\ dz\ r\ \ dr\ d\theta$$

6.
$$\int_0^{2\pi}\int_0^{1}\int_{-1/2}^{1/2} \left(r^2\sin^2\theta + z^2\right)\ dz\ r\ \ dr\ d\theta$$

In Exercises 7 - 10, evaluate the integrals.

7.
$$\int_0^{2\pi}\int_0^{3}\int_0^{z/3} r^3\ dr\quad dz\ d\theta$$

8
$$\int_{-1}^{1}\int_0^{2\pi}\int_0^{1+\cos\theta} 4r\ dr\ \ d\theta\ dz$$

9.
$$\int_0^{1}\int_0^{\sqrt{z}}\int_0^{2\pi} \left(r^2\cos^2\theta + z^2\right) r\ d\theta\ \ dr\ dz$$

10.
$$\int_0^2 \int_{r-2}^{\sqrt{4-r^2}} \int_0^{2\pi} (r\sin\theta + 1)\, r \; d\theta \; dz \; dr$$

Evaluate the spherical coordinate integrals in Exercises 11-16

11.
$$\int_0^{\pi} \int_0^{\pi} \int_0^{2\sin\phi} \rho^2 \sin\phi \; d\rho \; d\phi \; d\theta$$

12.
$$\int_0^{2\pi} \int_0^{\pi/4} \int_0^{2} (\rho\cos\phi)\rho^2 \sin\phi \; d\rho \; d\phi \; d\theta$$

13.
$$\int_0^{2\pi} \int_0^{\pi} \int_0^{(1-\cos\phi)/2} \rho^2 \sin\phi \; d\rho \; d\phi \; d\theta$$

14.
$$\int_0^{3\pi/2} \int_0^{\pi} \int_0^{1} 5\rho^3 \sin^3\phi \; d\rho \; d\phi \; d\theta$$

15.
$$\int_0^{2\pi} \int_0^{\pi/3} \int_{\sec\phi}^{2} 3\rho^2 \sin\phi \; d\rho \; d\phi \; d\theta$$

16.
$$\int_0^{2\pi} \int_0^{\pi/4} \int_0^{\sec\phi} (\rho\cos\phi)\, \rho^2 \sin\phi \; d\rho \; d\phi \; d\theta$$

Evaluate the integrals in Problems 17-20.

17.
$$\int_0^{2} \int_{-\pi}^{0} \int_{\pi/4}^{\pi/2} \rho^3 \sin 2\phi \; d\phi \; d\theta \; d\rho$$

18.
$$\int_{\pi/6}^{\pi/3} \int_{\csc\phi}^{2\csc\phi} \int_0^{2\pi} \rho^2 \sin\phi \; d\theta \; d\rho \; d\phi$$

19.

$$\int_0^1 \int_0^{\pi} \int_0^{\pi/4} 12\rho \sin^3 \phi \; d\phi \; d\theta \; d\rho$$

20.

$$\int_{\pi/6}^{\pi/2} \int_{-\pi/2}^{\pi/2} \int_{\csc \phi}^{2} 5\rho^4 \sin^3 \phi \; d\rho \; d\theta \; d\phi$$

In Exercises 19 - 28, find the volumes of the indicated solids.

19. Find the volume of the portion of the solid sphere $\rho \leq a$ that lies between the cones $\phi = \pi/3$ and $\phi = 2\pi/3$.

20. Find the volume of the solid enclosed by the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 2$.

21. Find the volume of the region bounded below by the plane $z = 0$, laterally by the cylinder $x^2 + y^2 = 1$, and above by the paraboloid $z = x^2 + y^2$.

22. Find the volume of the region bounded below by the paraboloid $z = x^2+y^2$, laterally by the cylinder $x^2+y^2 = 1$, and above by the paraboloid $z = x^2+y^2+1$.

23. Find the volume of the region bounded above by the sphere $x^2+y^2+z^2 = 2$ and below by the paraboloid $z = x^2 + y^2$.

24. Find the volume of the region cut from the solid cylinder $x^2 + y^2 \leq 1$ by the sphere $x^2 + y^2 + z^2 = 4$.

25. Find the volume of the region bounded above by the paraboloid $z = 9 - x^2 - y^2$, below by the $xy-$ plane and lying outside the cylinder $x^2 + y^2 = 1$