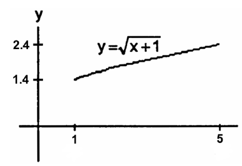
Your Name / Adınız - Soyadınız (**time:60**) Signature / İmza

Student ID # / Öğrenci No (**mavi tükenmez!**)

Problem	1	2	3	4	Total
Points:	15	30	25	30	100
Score:					

1. (15 points) Find the area of the surface generated by revolving $y=\sqrt{x+1}$, $1 \le x \le 5$ about the x-axis.



Solution:

$$y = \sqrt{x+1} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x+1}} \Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{4(x+1)} \Rightarrow S = \int_1^5 2\pi\sqrt{x+1}\sqrt{1+\frac{1}{4(x+1)}}\,dx$$

$$S = 2\pi\int_1^5 \sqrt{(x+1)+\frac{1}{4}}\,dx$$

$$= 2\pi\int_1^5 \sqrt{x+\frac{5}{4}}\,dx = 2\pi\left[\frac{2}{3}\left(x+\frac{5}{4}\right)^{3/2}\right]_1^5 = \frac{4\pi}{3}\left[\left(5+\frac{5}{4}\right)^{3/2} - \left(1+\frac{5}{4}\right)^{3/2}\right]$$

$$= \frac{4\pi}{3}\left[\left(\frac{25}{4}\right)^{3/2} - \left(\frac{9}{4}\right)^{3/2}\right] = \frac{4\pi}{3}\left(\frac{5^3}{2^3}-\frac{3^3}{2^3}\right)$$

$$\longrightarrow \quad S = \frac{\pi}{6}(125-27) = \frac{98\pi}{6} = \boxed{\frac{49\pi}{3}}$$

p.335, pr.16

2. (a) (15 Points) $\lim_{x\to 0^+} \frac{\ln(e^x-1)}{\ln x} = ?$

Solution:

$$\lim_{x\to 0^+} \frac{\ln(e^x-1)}{\ln x} = \lim_{x\to 0^+} \frac{\frac{e^x}{e^x-1}}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{xe^x}{e^x-1} = \lim_{x\to 0^+} \frac{e^x+xe^x}{e^x} = \frac{1+0}{1} = \boxed{1}.$$

p.402, pr.34

(b) (15 Points) The region between the curve $y=\frac{1}{x^2}$ and the x-axis from $x=1/2$ to $x=2$ is revolved about the y-axis to generate a solid. Find the volume of the solid.

Solution: By using the shell method (kabuk metodu), we have

$$V = 2\pi\int_{1/2}^2 x\,\frac{1}{x^2}\,dx = 2\pi\int_{1/2}^2 \frac{1}{x}\,dx = 2\pi\left[\ln|x|\right]_{1/2}^2 = 2\pi\left(\ln 2 - \ln\frac{1}{2}\right) = 2\pi\,(2\ln 2) = \pi\left(\ln 2^4\right) = \boxed{\pi\,(\ln 16)}$$

p.376, pr.75

3. (a) (10 Points) $\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = ?$

Solution:

$$\tan\left(\sin^{-1}\left(-\frac{1}{2}\right)\right) = \tan\left(-\frac{\pi}{6}\right) = \boxed{-\frac{1}{\sqrt{3}}}$$

p.413, pr.11

(b) (15 Points) $\int_0^2 \frac{dt}{8+2t^2} = ?$

Solution: Let $u = \sqrt{2}t$ and so $du = \sqrt{2}\,dt$; $t = 0 \Rightarrow u = 0$, $t = 2 \Rightarrow u = 2\sqrt{2}$

$$\begin{aligned}
\int_0^2 \frac{dt}{8+2t^2} &= \frac{1}{\sqrt{2}}\int_0^{2\sqrt{2}} \frac{du}{8+u^2} \\
&= \left[\frac{1}{2}\cdot\frac{1}{\sqrt{8}}\tan^{-1}\frac{u}{\sqrt{8}}\right]_0^{2\sqrt{2}} \\
&= \frac{1}{4}\left(\tan^{-1}\frac{2\sqrt{2}}{\sqrt{8}} - \tan^{-1} 0\right) \\
&= \frac{1}{4}\left(\tan^{-1}(1) - \tan^{-1}(0)\right) = \frac{1}{4}\left(\frac{\pi}{4} - 0\right) = \boxed{\frac{\pi}{16}}
\end{aligned}$$

p.413, pr.51

4. (a) (15 Points) $\displaystyle\int_0^{\ln 2} \tanh(2x)\,dx = ?$

Solution: Let $u = \cosh(2x)$ and so $du = 2\sinh(2x)\,dx$. If $x = 0$, then $u = \cosh(2\cdot 0) = 1$ and if $x = \ln 2$, then $u = \cosh(2\ln 2) = \cosh(\ln 4) = \dfrac{e^{\ln 4} + e^{-\ln 4}}{2} = \dfrac{4+\frac{1}{4}}{2} = \dfrac{17}{8}$

$$\begin{aligned}
\int_0^{\ln 2} \tanh(2x)\,dx &= \int_0^{\ln 2} \frac{\sinh(2x)}{\cosh(2x)}\,dx = \frac{1}{2}\int_1^{17/8} \frac{1}{u}\,du \\
&= \left[\frac{1}{2}\ln|u|\right]_1^{17/8} = \frac{1}{2}\left[\ln\frac{17}{8} - \ln 1\right] = \boxed{\frac{1}{2}\ln\frac{17}{8}}.
\end{aligned}$$

p.422, pr.52

(b) (15 Points) Find the length of the graph of $y = \dfrac{1}{2}\cosh(2x)$ from $x = 0$ to $x = \ln\sqrt{5}$.

Solution: $y = \dfrac{1}{2}\cosh(2x) \Rightarrow y' = \sinh(2x)$. Then we have

$$\begin{aligned}
L &= \int_0^{\ln\sqrt{5}} \sqrt{1 + (\sinh(2x))^2}\,dx = \int_0^{\ln\sqrt{5}} \sqrt{\cosh^2(2x)}\,dx = \int_0^{\ln\sqrt{5}} \cosh(2x)\,dx \\
&= \left[\frac{1}{2}\sinh(2x)\right]_0^{\ln\sqrt{5}} \\
&= \left[\frac{1}{2}\left(\frac{e^{2x} - e^{-2x}}{2}\right)\right]_0^{\ln\sqrt{5}} = \frac{1}{4}\left(5 - \frac{1}{5}\right) = \boxed{\frac{6}{5}}
\end{aligned}$$

p.423, pr.81