Mat 113 Yaz 2017 **Yarıyılsonu Sınavı** 3 Ağustos 2017

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Student ID # / Öğrenci No (**mavi tükenmez!**)

Soru	1	2	3	4	Toplam
Puan	32	20	25	23	100
Sonuç					

Sınav süresi **75 dakika**. **(Cep telefonlarınızı kapatın ve kürsüye bırakın!)**. Sınav boyunca dersle ilgili not, defter kitap vb. kapalı tutulacak. Cevaplarınızı **AÇIKLAMALISINIZ**. Yeterince açıklanmamış cevaplar –sonuç doğru olsa bile– ya hiç puan alamayacak ya da çok az puan alacak.

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1. (a) (8 Puan) $\int \frac{e^r}{1+e^r}\,dr$ integralini bulunuz.

Solution: First let $u = 1+e^r$. Then $du = e^r\,dr$. Therefore

$$\int \frac{e^r}{1+e^r}\,dr = \int \underbrace{\frac{1}{1+e^r}}_{1/u}\underbrace{e^r\,dr}_{du}$$
$$= \int \frac{1}{u}\,du$$
$$= \ln|1+e^r| + C = \boxed{\ln(1+e^r)+C}$$

p.385, pr.49

(b) (7 Puan) $y = \int_{\sec x}^{2} \frac{1}{t^2+1}\,dt$ ise $\frac{dy}{dx}$ türevini bulunuz.

Solution: First let $u = \sec x$ and so $du = \sec x \tan x\,dx$. Now we have

$$\frac{dy}{dx} = \frac{d}{dx}\left(\int_{\sec x}^{2} \frac{dt}{1+t^2}\right)$$
$$= \frac{d}{dx}\left(-\int_{2}^{\sec x} \frac{dt}{1+t^2}\right)$$
$$= -\frac{d}{dx}\left(\int_{2}^{u} \frac{dt}{1+t^2}\right)$$
$$= -\frac{d}{du}\left(\int_{2}^{u} \frac{dt}{1+t^2}\right)\frac{du}{dx}$$
$$= -\frac{1}{1+u^2}\sec x \tan x$$
$$= -\frac{1}{1+\sec^2 x}\sec x\tan x = \boxed{-\frac{\sec x\tan x}{1+\sec^2 x}}$$

page 303, problem 80

(c) (7 Puan) $\lim_{x\to 0} \frac{8x^2}{\cos x - 1}$ limitini bulunuz.

L'Hôpital kuralı gerekliyse kullanabilirsiniz.

Solution: We can apply l'Hôpital's Rule twice to evaluate this limit. We have

$$\lim_{x\to 0}\frac{8x^2}{\cos x-1} = \lim_{x\to 0}\frac{\frac{d}{dx}(8x^2)}{\frac{d}{dx}(\cos x-1)}$$
$$= \lim_{x\to 0}\frac{16x}{-\sin x}$$
$$= \lim_{x\to 0}\frac{16}{-\cos x} = \frac{16}{(-\cos 0)} = \boxed{-16}$$

p.55 pr.32

(d) (10 Puan) $\int \frac{dx}{2\sqrt{x}+2x}$ integralini bulunuz.

Solution: First notice the following.

$$\int \frac{dx}{2\sqrt{x}+2x} = \int \frac{dx}{2\sqrt{x}(1+\sqrt{x})}.$$

Therefore if we let $u = 1+\sqrt{x}$, then $du = \frac{1}{2\sqrt{x}}\,dx$ and so we have

$$\int \frac{dx}{2\sqrt{x}+2x} = \int \frac{1}{1+\sqrt{x}}\underbrace{\frac{1}{2\sqrt{x}}\,dx}_{du}$$
$$= \int \frac{1}{u}\,du$$
$$= \ln|u| + c = \ln|1+\sqrt{x}| + c$$
$$= \boxed{\ln(1+\sqrt{x})+c}$$

page 376, problem 53

2. (a) (10 Puan) $-2 \le x \le 2$ ise $y = x^2$ ve $y = x + 2$ ile sınırlı bölgenin alanını bulunuz.

Solution: One shall find this area by integrating with respect to x since this is easier than y. Therefore we have

$$A = \int_{-2}^{-1} \Big(\underbrace{x^2}_{\text{upper curve}} - \underbrace{(x+2)}_{\text{lower curve}} \Big)\, dx + \int_{-1}^{2} \Big(\underbrace{(x+2)}_{\text{upper curve}} - \underbrace{x^2}_{\text{lower curve}} \Big)\, dx$$

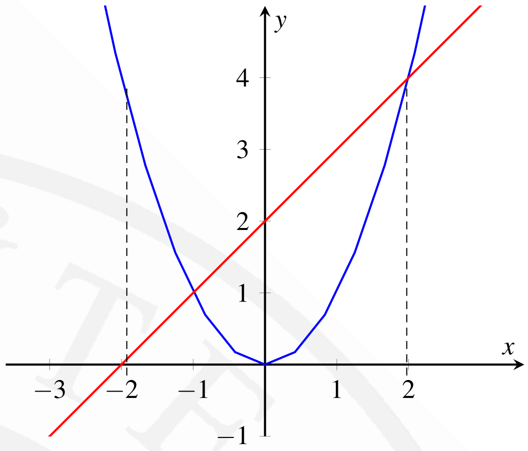
$$= \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x \right]_{-2}^{-1} + \left[\frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_{-1}^{2}$$

$$= -\frac{1}{3} - \frac{1}{2} + 2 - \left(-\frac{8}{3} - 2 + 4 \right) + \left(2 + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} - 2 + \frac{1}{3} \right)$$

$$= 7 - \frac{2}{3}$$

$$= \boxed{\frac{19}{3}}$$

p.298, pr.32



(b) (10 Puan) Kabuk yöntemi kullanarak, $y = 2 - x^2$ ile $y = x^2$ sınırlı bölgenin y-ekseni etrafında döndürülmesiyle oluşan cismin hacmini bulunuz.

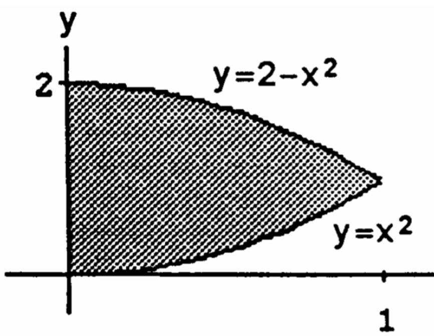
Solution: If $0 \le x \le 2$, then a vertical strip of the given region "at“ x has length $-(x^2 - 2x)$ and moves around a circle of radius $2 - x$, so the volume generated rotation of that region around $x = 2$ is

$$V = \int_a^b 2\pi \begin{pmatrix} \text{shell} \\ \text{radius} \end{pmatrix} \begin{pmatrix} \text{shell} \\ \text{height} \end{pmatrix} dx = 2\pi \int_0^1 x[(2-x^2) - x^2]\, dx = 2\pi \int_0^1 x(2-x^2)\, dx$$

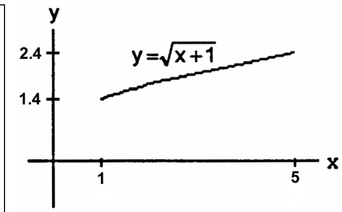
$$= 4\pi \int_0^1 (x - x^3)\, dx$$

$$= 4\pi \left[\frac{1}{2}x^2 - \frac{1}{4}x^4 \right]_0^1 = 4\pi \left[\frac{1}{2} - \frac{1}{4} \right] = \boxed{\pi}$$

p.325, pr.10



3. (a) (12 Puan) $y=\sqrt{x+1}$, $1\le x\le 5$ eğrisinin x-ekseni etrafında döndürülmesiyle oluşan yüzeyin alanını bulunuz.



Solution:

$$y=\sqrt{x+1}\Rightarrow \frac{dy}{dx}=\frac{1}{2\sqrt{x+1}}\Rightarrow \left(\frac{dy}{dx}\right)^2=\frac{1}{4(x+1)}\Rightarrow S=\int_1^5 2\pi\sqrt{x+1}\sqrt{1+\frac{1}{4(x+1)}}\,dx$$

$$S=2\pi\int_1^5\sqrt{(x+1)+\frac{1}{4}}\,dx$$

$$=2\pi\int_1^5\sqrt{x+\frac{5}{4}}\,dx=2\pi\left[\frac{2}{3}\left(x+\frac{5}{4}\right)^{3/2}\right]_1^5=\frac{4\pi}{3}\left[\left(5+\frac{5}{4}\right)^{3/2}-\left(1+\frac{5}{4}\right)^{3/2}\right]$$

$$=\frac{4\pi}{3}\left[\left(\frac{25}{4}\right)^{3/2}-\left(\frac{9}{4}\right)^{3/2}\right]=\frac{4\pi}{3}\left(\frac{5^3}{2^3}-\frac{3^3}{2^3}\right)$$

$$\longrightarrow S=\frac{\pi}{6}(125-27)=\frac{98\pi}{6}=\boxed{\frac{49\pi}{3}}$$

p.335, pr.16

(b) (13 Puan) $f(x)=x^2-4x-5$, $x>2$ olsun. $(df^{-1})/dx$ in değerini $x=0=f(5)$ noktasında bulunuz

Solution:

$$\frac{df}{dx}=\frac{d}{dx}(x^2-4x-5)=2x-4$$

$$\left[\frac{df^{-1}}{dx}\right]_{x=f(5)}=\left[\frac{1}{\frac{df}{dx}}\right]_{x=f(5)}$$

$$=\frac{1}{(2)(5)-4}=\boxed{\frac{1}{6}}.$$

p.368, pr.42

4. $y = -x^4 + 6x^2 - 4$ veriliyor. Ayr ca $y' = -4x^3 + 12x = -4x(x+\sqrt{3})(x-\sqrt{3})$ ve $y'' = -12x^2 + 12 = -12(x+1)(x-1)$ kabul edebilirsiniz.

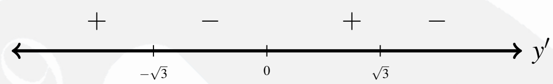
(a) (5 Puan) *Tanım kümesini* yazınız.

Solution: This is a polynomial function so the domain is $(-\infty, \infty)$.

p.211, pr.18

(b) (5 Puan) Fonksiyonun *arttığı* ve *azaldığı aralıkları* bulunuz. *Yerel maksimum* ve *minimum* değerleri bulunuz.

Solution: The first derivative is $y' = -4x^3 + 12x = -4x(x+\sqrt{3})(x-\sqrt{3})$. The three critical numbers are $x = 0$ and $x = \pm\sqrt{3}$. This brings about four intervals, namely $(-\infty, -\sqrt{3})$, $(-\sqrt{3}, 0)$, $(0, \sqrt{3})$ and $(\sqrt{3}, \infty)$. On the intervals $(-\infty, -\sqrt{3})$ and $(0, \sqrt{3})$ graph is increasing and on the intervals $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$ graph is decreasing. Furthermore, the points $(-\sqrt{3}, 5)$ and $(\sqrt{3}, 5)$ are points of absolute maximum and $(0, -4)$ is a point of local minimum.



p.211, pr.18

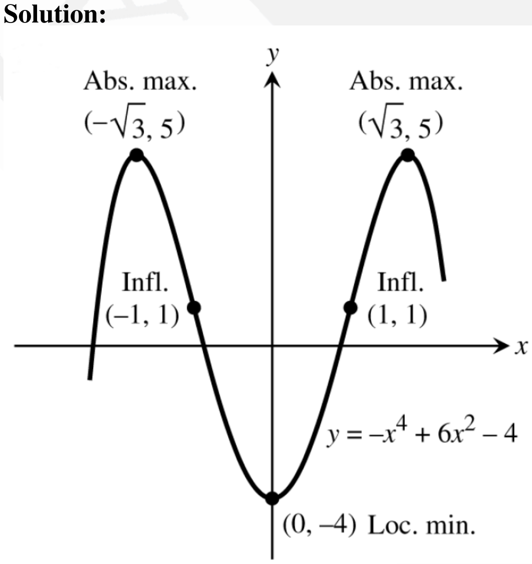
(c) (5 Puan) Grafiğin *aşağı konkav* ve *yukarı konkav* olduğu *aralıkları* bulunuz. Varsa, *büküm noktalarını* yazınız.

Solution: Now the second derivative is $y'' = -12(x+1)(x-1)$ which is zero only when $x = \pm 1$. So this splits the real line into three subintervals: $(-\infty, -1)$, $(-1, +1)$ and $(+1, \infty)$. Since over $(-\infty, 1) \cup (1, \infty)$ second derivative is negative and and on $(-1, +1)$, y'' is positive, we conclude that the graph is concave up on $(-1, +1)$ and concave down on $(-\infty, -1)$ and $(1, \infty)$. Moreover, $(-1, 1)$ and $(1, 1)$ are points of inflection.

p.211, pr.18

(d) (8 Puan) Fonksiyonun *grafiğini çiziniz*. Dönüm ve büküm noktalarını belirtiniz.

Solution:



p.211, pr.18